False Discovery Type Procedures: caveats to reproducibility–its all about dispersion

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Outline

1. Introduction
   - BH-FDR Procedure: A review & Its properties
   - Main theoretical results

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   - Main theoretical results
2. Other Methods for FDP control
   - The case for controlling the right tail of the FDP
   - Romano's procedure
   - The BH-CLT procedure

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- R_m is the number declared significant.

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- The connection with the empirical CDF is used to prove a central limit theorem. Non-optionally Stopped Brownian Bridge.

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- $G(0) = 0, G(1) = 1.$
- The minimal point of intersection of G with its concave hull, $\tilde{G}$ occurs at a value $\tilde{u} = \inf\{u : G(u) = \tilde{G}(u)\}$ that is less than 1.

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Some related facts

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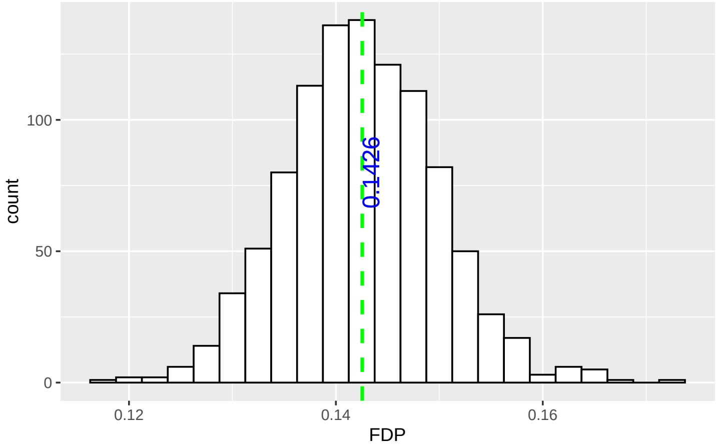
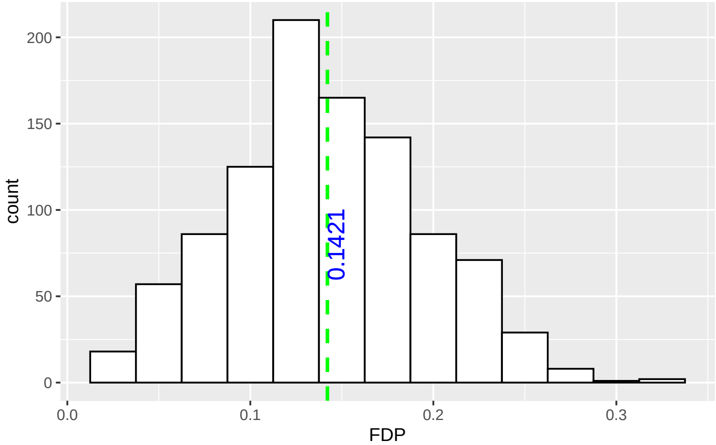
- $\pi_1 \doteq \mathbb{E}[T_m/M_m]$ is the *Average Power*

Distribution of the FDP for 50k Simultaneous Tests



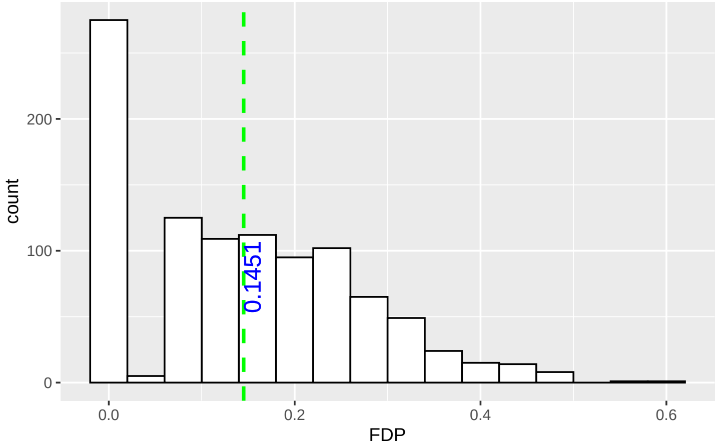
m=50k, eff sz=0.5, p1=0.05, n=113, avg pwr= 0.85, FDR cntrld at alpha=0.15, 1000 sim reps.

Distribution of the FDP for 1000 Simultaneous Tests



m=1000, eff sz=0.5, p1=0.05, n=113, avg pwr= 0.85, FDR cntrld at alpha=0.15, 1000 sim reps.

Distribution of the FDP for 200 Simultaneous Tests



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- In addition to all of the above 4 conditions, assume that G is differentiable at $\gamma\alpha$ and $\dot{G}(\gamma\alpha) < 1/\alpha$, then:

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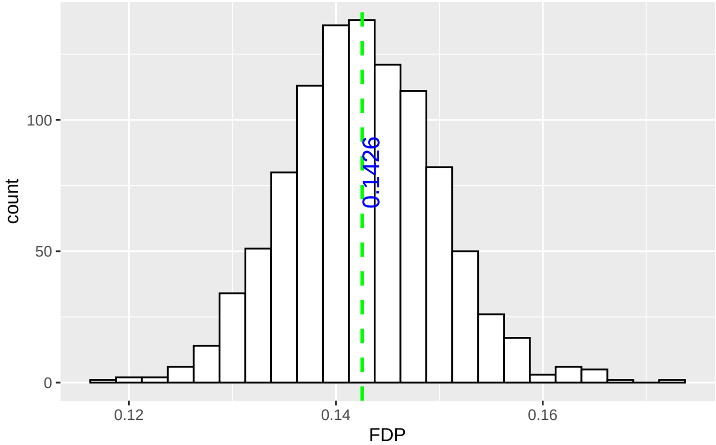
\text{and } \sigma^2 &= p_1^{-2} p_1\pi_1(1 - p_1\pi_1) + \ldots
\end{aligned}$$

Outline

The BH-FDR Procedure Controls the $FDR = \mathbb{E}[FDP] \leq \alpha$



m=50k, eff sz=0.5, p1=0.05, n=113, avg pwr= 0.85, FDR cntrld at alpha=0.15, 1000 sim reps.

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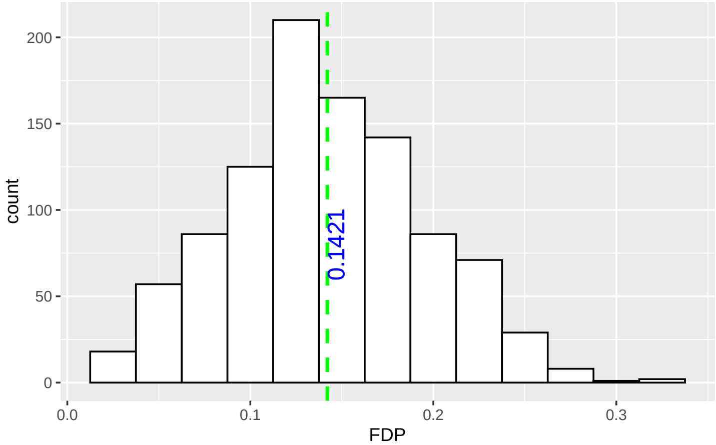

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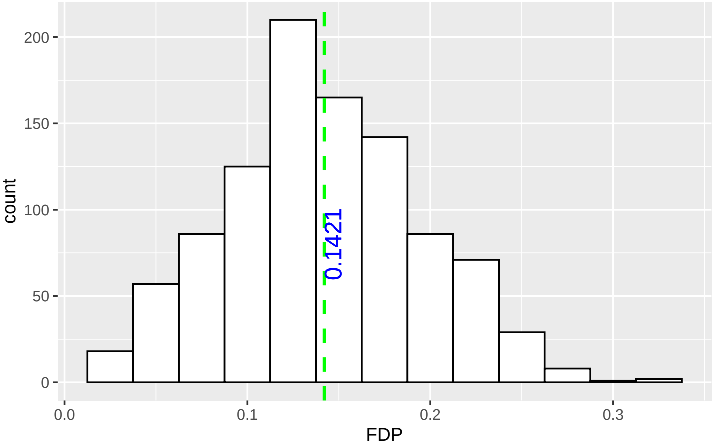

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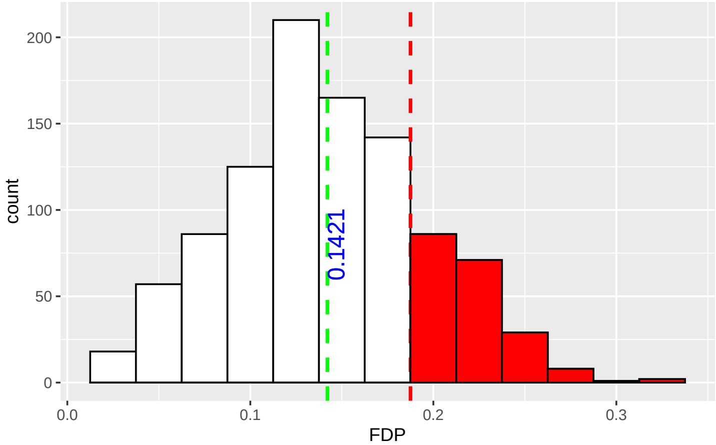

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$$V_m^{\dagger} = \sum_{i=1}^{R_m^{\dagger}} (1 - \xi_{(i:m)}), \quad T_m^{\dagger} = \sum_{i=1}^{R_m^{\dagger}} \xi_{(i:m)}.$$

Romano procedure versus BH-FDR procedure

$$\text{BH-FDR: } R_m = \max\left\{ i : P_{(i:m)} \le \alpha i/m \right\}$$

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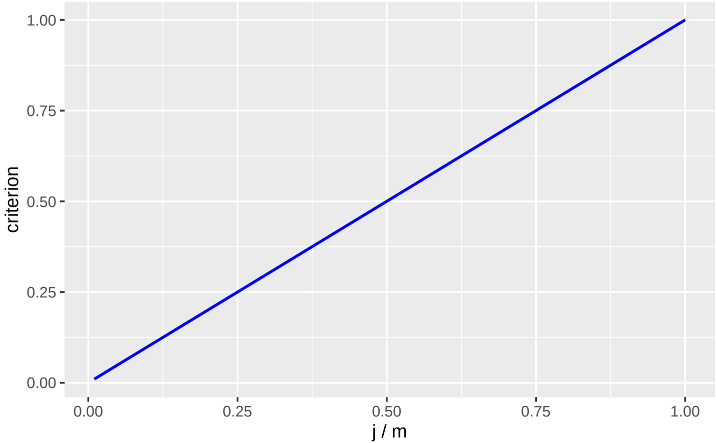
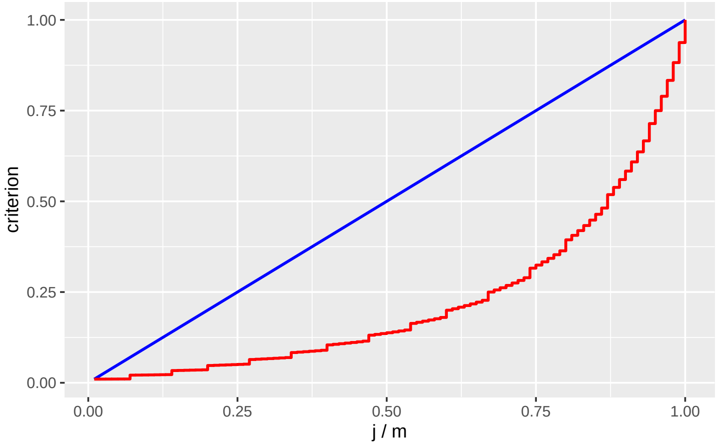
- No limit law for $T^{\dagger}/M_m$, so no closed form power function.

Romano procedure versus BH-FDR procedure



Romano ψ_m and BHFDR identity function versus u, and Bonferroni, dashed, m=100, alpha=0.15

Romano procedure versus BH-FDR procedure



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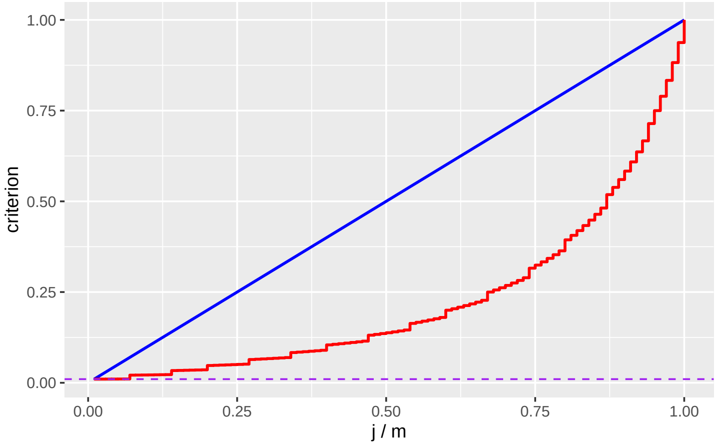

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Romano procedure versus BH-FDR procedure

X_i	ξ_i	P_i	$\alpha i/m$	$\alpha\psi_m(i,\alpha)$	$P_i \leq (4)$	$P_i \leq (5)$
4.32	1	0.000024	0.000750	0.000750	1	1
4.21	1	0.000036	0.001500	0.000754	1	1
4.13	1	0.000051	0.002250	0.000758	1	1
3.51	1	0.000536	0.003000	0.000761	1	1
3.41	1	0.000766	0.003750	0.000765	1	0
3.41	1	0.000768	0.004500	0.000769	1	1
3.05	1	0.002597	0.005250	0.001538	1	0
3.03	1	0.002760	0.006000	0.001546	1	0
2.98	1	0.003245	0.006750	0.001554	1	0
-2.69	0	0.007591	0.007500	0.001562	0	0
2.69	1	0.007652	0.008250	0.001571	1	0
2.46	0	0.014481	0.009000	0.001579	0	0
-2.21	0	0.028401	0.009750	0.001587	0	0
2.15	0	0.032578	0.010500	0.002381	0	0
-2.11	0	0.035854	0.011250	0.002394	0	0

Table 1: Comparing the BHFDR and Romano procedures

Modified Romano procedure

$$\text{BH-FDR: } R_m \quad = \quad \max\left\{i : P_{(i:m)} \leq \alpha i/m\right\}$$

Modified Romano procedure

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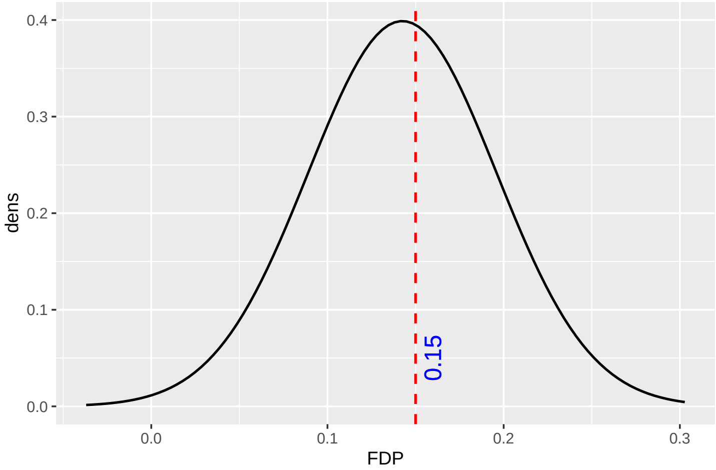
- We conjecture asymptotic equivalence.

Outline

The BH-CLT procedure



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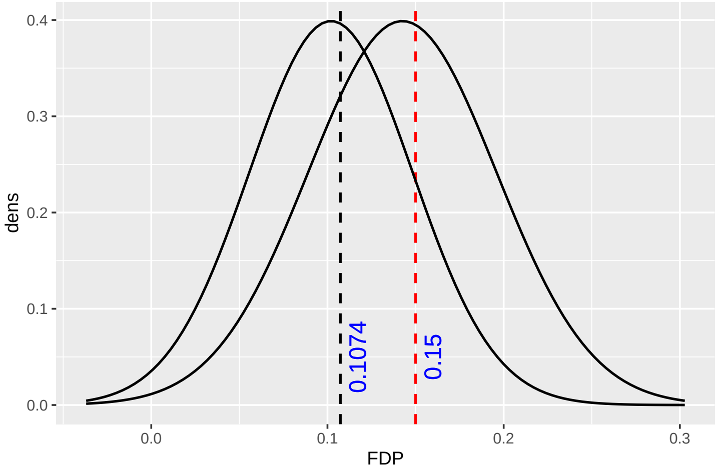

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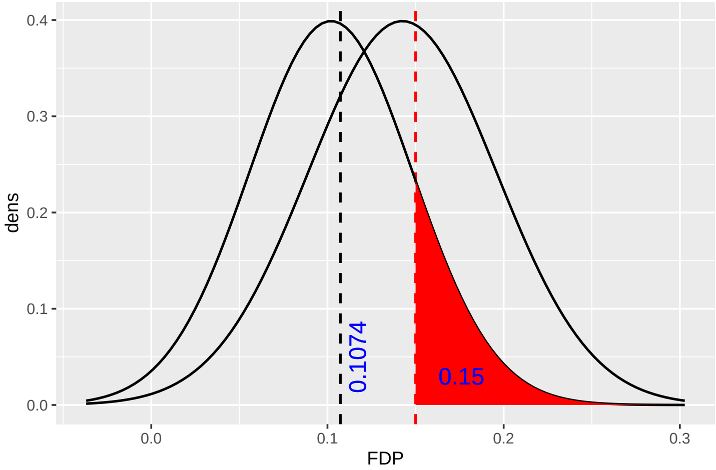

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- Corresponding limit laws.

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Let $\alpha_\ell^\star = p_0^{-1}\delta - \left(p_0 \sqrt{m}\right)^{-1} s_\ell^\star \Phi^{-1}(1-\alpha)$

The BH-CLT procedure

BHCLT(δ, α)

Let $\ell = 1, \epsilon_0 = 1$, and $\alpha_0^\star = \alpha$

While $(\epsilon_{\ell-1} > \text{tol})$

Let $\gamma_\ell^\star$, be the solution to $\gamma^\star = G\left(\gamma^\star \alpha_{\ell-1}^\star\right)$

Let $s_\ell^\star = \sqrt{p_0 \alpha_{\ell-1}^\star (1 - p_0 \alpha_{\ell-1}^\star \gamma_\ell^\star)/\gamma_\ell^\star}$

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  - Modified Romano procedure has a closed form power function but you need $mp_1 > 1$ or 2 for it to work
- BHCLT: requires adequate asymptotic approximation, $m \geq 50$. Makes a substantial difference if $StdErr[FDP]/\alpha > 0.10$ or so.

Outline

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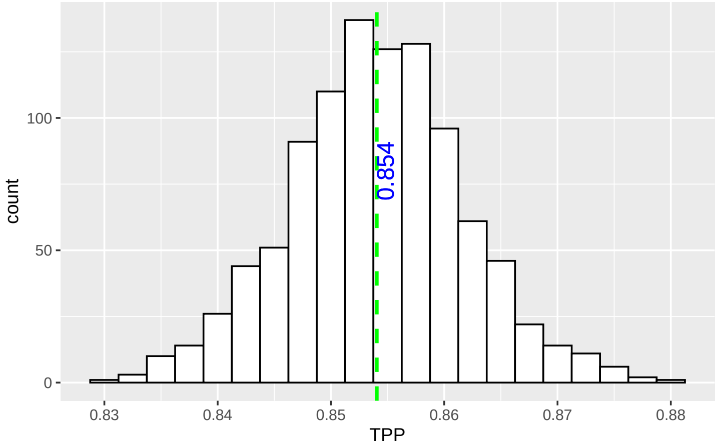
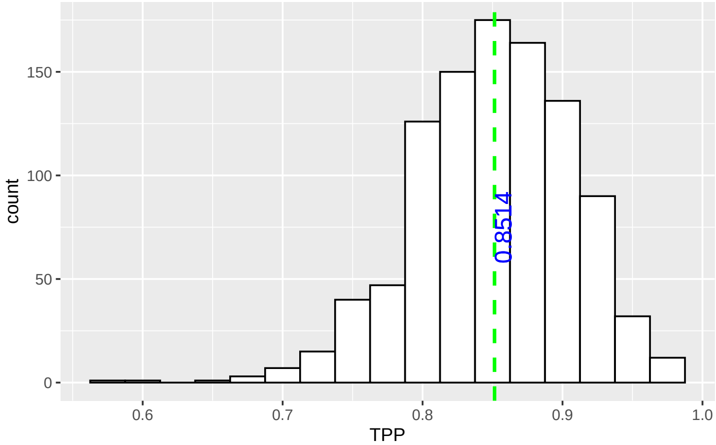
Distribution of the TPP for 50,000 simultaneous tests



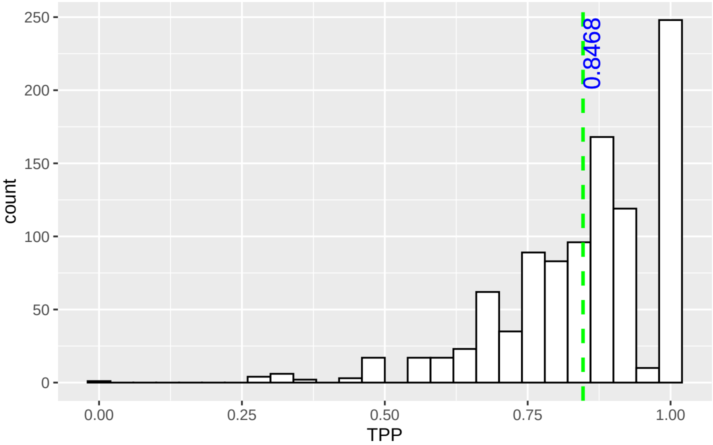
m=50k, eff sz=0.5, p1=0.05, n=113, avg pwr= 0.85, FDR cntrld at alpha=0.15, 1000 sim reps.

Distribution of the TPP for 1,000 simultaneous tests



m=1k, eff sz=0.5, p1=0.05, n=113, avg pwr= 0.85, FDR cntrld at alpha=0.15, 1000 sim reps.

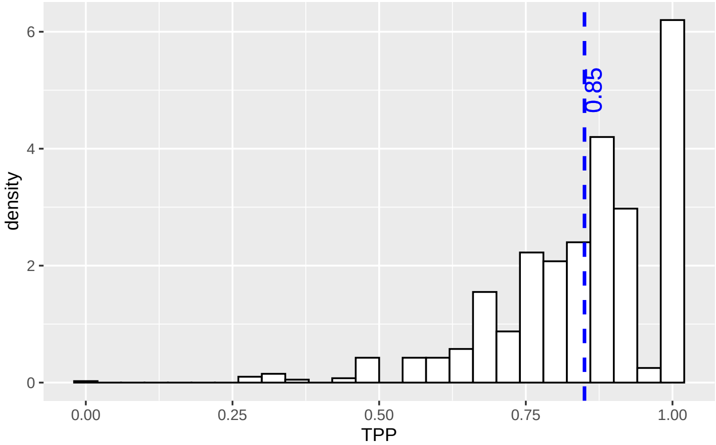
Distribution of the TPP for 200 simultaneous tests



m=200, eff sz=0.5, p1=0.05, n=113, avg pwr= 0.85, FDR cntrld at alpha=0.15, 1000 sim reps.

Outline

tp-TPP Power: the right tail of the TPP distribution



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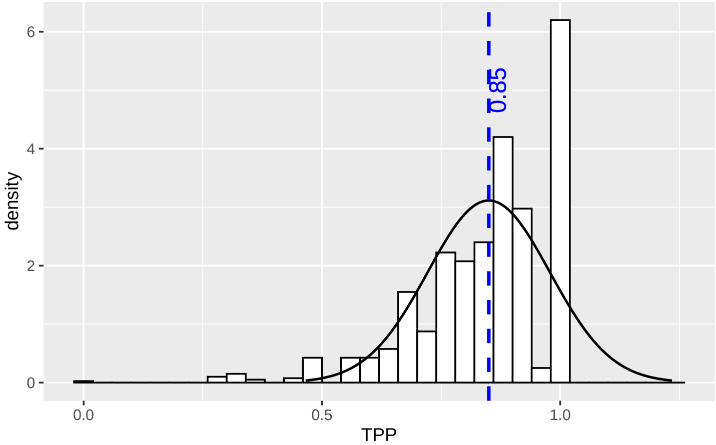

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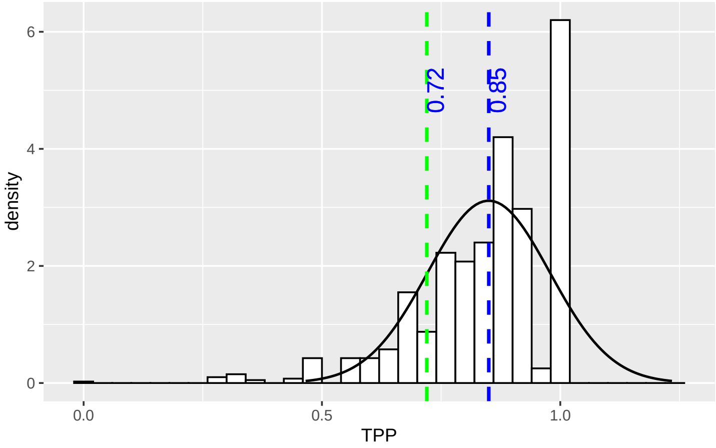

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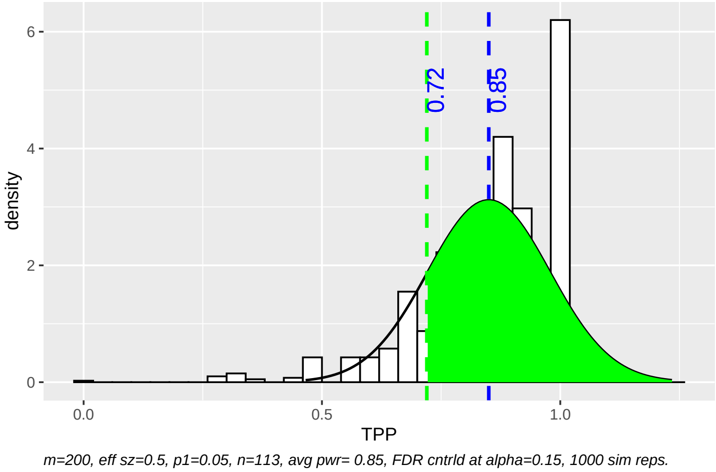

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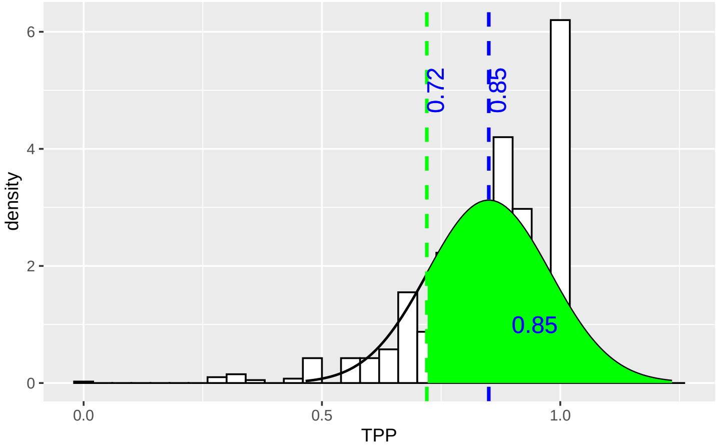

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Possibilities: FDP control method and definitin of power

FDP-Cntl-Mthd	Avg Pwr	tp-TPP Pwr
Romano	*	*
BHCLT	*	*
BH-FDR	*	*

Table 2: FDP-Control methods for two definitions of power

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  - Job took 1.66 ± 0.014 hours

Outline

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	68	CLT	0.15	0.143	0.039	0.039	0.915	0.873
0.7	0.10	60	FDR		0.136	0.105	0.105	0.916	0.956
0.7	0.25	49	FDR		0.112	0.000	0.000	0.913	0.982
0.7	0.30	46	FDR		0.105	0.000	0.000	0.907	0.869
0.8	0.05	53	CLT	0.15	0.142	0.032	0.032	0.918	0.921
0.8	0.10	46	FDR		0.135	0.099	0.099	0.913	0.916
0.8	0.25	38	FDR		0.113	0.000	0.000	0.915	0.988
0.8	0.30	36	FDR		0.105	0.000	0.000	0.911	0.970
0.9	0.05	42	CLT	0.15	0.142	0.033	0.033	0.917	0.897
0.9	0.10	37	FDR		0.135	0.103	0.103	0.916	0.943
0.9	0.25	30	FDR		0.112	0.000	0.000	0.912	0.964
0.9	0.30	29	FDR		0.105	0.000	0.000	0.915	0.996
1.0	0.05	35	CLT	0.15	0.143	0.037	0.037	0.923	0.953
1.0	0.10	30	FDR		0.135	0.106	0.106	0.914	0.923
1.0	0.25	25	FDR		0.113	0.000	0.000	0.918	0.997
1.0	0.30	24	FDR		0.105	0.000	0.000	0.919	0.999
1.1	0.05	29	CLT	0.15	0.143	0.039	0.039	0.920	0.939
1.1	0.10	25	FDR		0.135	0.095	0.095	0.913	0.905
1.1	0.25	21	FDR		0.113	0.000	0.000	0.920	1.000
1.1	0.30	20	FDR		0.105	0.000	0.000	0.918	1.000

Table 3: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=10000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.15	0.143	0.036	0.036	0.858	0.008
0.7	0.10	51	FDR		0.136	0.111	0.111	0.852	0.000
0.7	0.25	42	FDR		0.113	0.000	0.000	0.858	0.000
0.7	0.30	40	FDR		0.105	0.000	0.000	0.857	0.000
0.8	0.05	45	CLT	0.15	0.142	0.039	0.039	0.851	0.001
0.8	0.10	39	FDR		0.136	0.098	0.098	0.847	0.000
0.8	0.25	33	FDR		0.113	0.000	0.000	0.865	0.000
0.8	0.30	31	FDR		0.105	0.000	0.000	0.859	0.000
0.9	0.05	36	CLT	0.15	0.143	0.039	0.039	0.852	0.003
0.9	0.10	32	FDR		0.134	0.080	0.080	0.859	0.001
0.9	0.25	26	FDR		0.113	0.000	0.000	0.860	0.000
0.9	0.30	25	FDR		0.105	0.000	0.000	0.863	0.000
1.0	0.05	30	CLT	0.15	0.142	0.026	0.026	0.860	0.010
1.0	0.10	26	FDR		0.135	0.094	0.094	0.856	0.000
1.0	0.25	21	FDR		0.113	0.000	0.000	0.854	0.000
1.0	0.30	20	FDR		0.105	0.000	0.000	0.854	0.000
1.1	0.05	25	CLT	0.15	0.142	0.042	0.042	0.858	0.004
1.1	0.10	22	FDR		0.136	0.111	0.111	0.861	0.001
1.1	0.25	18	FDR		0.113	0.000	0.000	0.865	0.000
1.1	0.30	17	FDR		0.105	0.000	0.000	0.860	0.000

Table 4: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=10000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	69	CLT	0.15	0.143	0.097	0.097	0.919	0.855
0.7	0.10	60	CLT	0.15	0.135	0.154	0.179	0.916	0.885
0.7	0.25	49	FDR		0.112	0.000	0.000	0.913	0.925
0.7	0.30	47	FDR		0.105	0.000	0.000	0.913	0.934
0.8	0.05	54	CLT	0.15	0.142	0.092	0.092	0.924	0.909
0.8	0.10	47	CLT	0.15	0.135	0.139	0.152	0.920	0.940
0.8	0.25	38	FDR		0.112	0.000	0.000	0.914	0.948
0.8	0.30	36	FDR		0.105	0.000	0.000	0.911	0.925
0.9	0.05	43	CLT	0.15	0.143	0.112	0.112	0.924	0.901
0.9	0.10	37	CLT	0.15	0.135	0.151	0.171	0.916	0.890
0.9	0.25	30	FDR		0.113	0.000	0.000	0.912	0.923
0.9	0.30	29	FDR		0.105	0.000	0.000	0.915	0.969
1.0	0.05	35	CLT	0.15	0.141	0.085	0.085	0.923	0.893
1.0	0.10	31	CLT	0.15	0.135	0.149	0.171	0.924	0.969
1.0	0.25	25	FDR		0.112	0.000	0.000	0.918	0.980
1.0	0.30	24	FDR		0.105	0.000	0.000	0.918	0.986
1.1	0.05	29	CLT	0.15	0.141	0.090	0.090	0.919	0.846
1.1	0.10	26	CLT	0.15	0.135	0.151	0.175	0.926	0.974
1.1	0.25	21	FDR		0.113	0.000	0.000	0.920	0.988
1.1	0.30	20	FDR		0.105	0.000	0.000	0.918	0.989

Table 5: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=5000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.15	0.143	0.101	0.101	0.857	0.042
0.7	0.10	51	CLT	0.15	0.134	0.128	0.159	0.851	0.004
0.7	0.25	42	FDR		0.112	0.000	0.000	0.858	0.000
0.7	0.30	40	FDR		0.105	0.000	0.000	0.857	0.000
0.8	0.05	45	CLT	0.15	0.142	0.086	0.086	0.851	0.022
0.8	0.10	39	CLT	0.15	0.135	0.144	0.175	0.846	0.000
0.8	0.25	33	FDR		0.113	0.000	0.000	0.866	0.000
0.8	0.30	31	FDR		0.105	0.000	0.000	0.858	0.000
0.9	0.05	36	CLT	0.15	0.143	0.103	0.103	0.851	0.016
0.9	0.10	32	CLT	0.15	0.136	0.163	0.195	0.860	0.009
0.9	0.25	26	FDR		0.113	0.000	0.000	0.860	0.000
0.9	0.30	25	FDR		0.105	0.000	0.000	0.863	0.000
1.0	0.05	30	CLT	0.15	0.142	0.092	0.092	0.860	0.043
1.0	0.10	26	CLT	0.15	0.134	0.156	0.169	0.855	0.003
1.0	0.25	21	FDR		0.112	0.000	0.000	0.854	0.000
1.0	0.30	20	FDR		0.106	0.000	0.000	0.853	0.000
1.1	0.05	25	CLT	0.15	0.144	0.115	0.115	0.858	0.042
1.1	0.10	22	CLT	0.15	0.135	0.160	0.186	0.860	0.005
1.1	0.25	18	FDR		0.113	0.000	0.000	0.864	0.000
1.1	0.30	17	FDR		0.105	0.000	0.000	0.861	0.000

Table 6: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=5000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	72	CLT	0.14	0.142	0.132	0.181	0.933	0.895
0.7	0.10	63	CLT	0.14	0.136	0.145	0.278	0.929	0.917
0.7	0.25	50	FDR		0.111	0.000	0.000	0.918	0.912
0.7	0.30	48	FDR		0.105	0.000	0.000	0.919	0.936
0.8	0.05	56	CLT	0.14	0.145	0.160	0.207	0.935	0.900
0.8	0.10	48	CLT	0.14	0.137	0.169	0.312	0.926	0.910
0.8	0.25	38	FDR		0.113	0.007	0.007	0.914	0.853
0.8	0.30	37	FDR		0.106	0.000	0.000	0.919	0.944
0.9	0.05	44	CLT	0.14	0.142	0.161	0.213	0.931	0.872
0.9	0.10	39	CLT	0.14	0.134	0.154	0.254	0.932	0.940
0.9	0.25	31	FDR		0.113	0.008	0.008	0.921	0.935
0.9	0.30	29	FDR		0.105	0.000	0.000	0.914	0.869
1.0	0.05	36	CLT	0.14	0.145	0.154	0.208	0.931	0.879
1.0	0.10	32	CLT	0.14	0.136	0.168	0.284	0.933	0.947
1.0	0.25	25	FDR		0.112	0.002	0.002	0.916	0.881
1.0	0.30	24	FDR		0.106	0.001	0.001	0.919	0.937
1.1	0.05	31	CLT	0.14	0.144	0.170	0.224	0.941	0.935
1.1	0.10	27	CLT	0.14	0.135	0.159	0.263	0.936	0.968
1.1	0.25	21	FDR		0.112	0.005	0.005	0.919	0.926
1.1	0.30	20	FDR		0.105	0.000	0.000	0.918	0.918

Table 7: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=2000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.14	0.140	0.141	0.206	0.857	0.146
0.7	0.10	51	CLT	0.14	0.135	0.166	0.283	0.852	0.049
0.7	0.25	42	FDR		0.112	0.007	0.007	0.857	0.006
0.7	0.30	40	FDR		0.105	0.001	0.001	0.857	0.003
0.8	0.05	45	CLT	0.14	0.143	0.152	0.232	0.850	0.112
0.8	0.10	39	CLT	0.14	0.136	0.169	0.302	0.848	0.031
0.8	0.25	33	FDR		0.113	0.004	0.004	0.865	0.020
0.8	0.30	31	FDR		0.105	0.001	0.001	0.859	0.003
0.9	0.05	36	CLT	0.14	0.143	0.164	0.215	0.851	0.112
0.9	0.10	32	CLT	0.14	0.135	0.155	0.279	0.860	0.071
0.9	0.25	26	FDR		0.112	0.006	0.006	0.860	0.012
0.9	0.30	25	FDR		0.104	0.000	0.000	0.863	0.012
1.0	0.05	30	CLT	0.14	0.143	0.149	0.210	0.861	0.179
1.0	0.10	26	CLT	0.14	0.134	0.125	0.267	0.854	0.045
1.0	0.25	21	FDR		0.112	0.008	0.008	0.855	0.003
1.0	0.30	20	FDR		0.104	0.000	0.000	0.852	0.000
1.1	0.05	25	CLT	0.14	0.142	0.144	0.196	0.855	0.122
1.1	0.10	22	CLT	0.14	0.136	0.151	0.295	0.860	0.058
1.1	0.25	18	FDR		0.113	0.009	0.009	0.865	0.020
1.1	0.30	17	FDR		0.105	0.000	0.000	0.860	0.006

Table 8: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=2000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	76	CLT	0.13	0.141	0.147	0.253	0.947	0.916
0.7	0.10	65	CLT	0.13	0.135	0.131	0.319	0.938	0.932
0.7	0.25	51	CLT	0.15	0.113	0.036	0.036	0.924	0.917
0.7	0.30	48	CLT	0.15	0.105	0.009	0.009	0.918	0.856
0.8	0.05	58	CLT	0.13	0.142	0.143	0.264	0.945	0.902
0.8	0.10	50	CLT	0.13	0.134	0.150	0.325	0.939	0.922
0.8	0.25	39	CLT	0.15	0.114	0.041	0.041	0.923	0.898
0.8	0.30	37	CLT	0.15	0.104	0.004	0.004	0.919	0.849
0.9	0.05	47	CLT	0.13	0.140	0.136	0.257	0.950	0.930
0.9	0.10	40	CLT	0.13	0.135	0.155	0.315	0.940	0.925
0.9	0.25	31	CLT	0.15	0.113	0.040	0.040	0.921	0.859
0.9	0.30	30	CLT	0.15	0.104	0.005	0.005	0.923	0.907
1.0	0.05	38	CLT	0.13	0.144	0.159	0.277	0.946	0.903
1.0	0.10	33	CLT	0.13	0.138	0.176	0.367	0.940	0.943
1.0	0.25	26	CLT	0.15	0.112	0.033	0.033	0.929	0.939
1.0	0.30	24	CLT	0.15	0.105	0.015	0.015	0.917	0.840
1.1	0.05	32	CLT	0.13	0.141	0.131	0.255	0.948	0.912
1.1	0.10	28	CLT	0.13	0.135	0.165	0.335	0.945	0.953
1.1	0.25	22	CLT	0.15	0.113	0.037	0.037	0.931	0.952
1.1	0.30	21	CLT	0.15	0.106	0.011	0.011	0.930	0.965

Table 9: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=1000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.13	0.141	0.152	0.269	0.855	0.227
0.7	0.10	51	CLT	0.13	0.134	0.154	0.323	0.852	0.117
0.7	0.25	42	CLT	0.15	0.113	0.042	0.042	0.858	0.043
0.7	0.30	40	CLT	0.15	0.105	0.015	0.015	0.856	0.027
0.8	0.05	45	CLT	0.13	0.141	0.153	0.276	0.850	0.210
0.8	0.10	39	CLT	0.13	0.135	0.148	0.332	0.846	0.097
0.8	0.25	33	CLT	0.15	0.113	0.048	0.048	0.865	0.067
0.8	0.30	31	CLT	0.15	0.104	0.007	0.007	0.858	0.041
0.9	0.05	36	CLT	0.13	0.141	0.140	0.255	0.851	0.208
0.9	0.10	32	CLT	0.13	0.138	0.147	0.344	0.860	0.171
0.9	0.25	26	CLT	0.15	0.112	0.049	0.049	0.860	0.054
0.9	0.30	25	CLT	0.15	0.106	0.010	0.010	0.864	0.051
1.0	0.05	30	CLT	0.13	0.145	0.151	0.301	0.859	0.258
1.0	0.10	26	CLT	0.13	0.135	0.158	0.328	0.856	0.135
1.0	0.25	21	CLT	0.15	0.113	0.046	0.046	0.855	0.030
1.0	0.30	20	CLT	0.15	0.105	0.012	0.012	0.853	0.023
1.1	0.05	25	CLT	0.13	0.142	0.148	0.278	0.856	0.241
1.1	0.10	22	CLT	0.13	0.136	0.161	0.355	0.859	0.157
1.1	0.25	18	CLT	0.15	0.112	0.035	0.035	0.865	0.079
1.1	0.30	17	CLT	0.15	0.106	0.008	0.008	0.860	0.039

Table 10: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=1000, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	81	CLT	0.11	0.141	0.154	0.320	0.960	0.920
0.7	0.10	69	CLT	0.12	0.135	0.151	0.364	0.953	0.934
0.7	0.25	52	CLT	0.15	0.113	0.108	0.108	0.929	0.888
0.7	0.30	49	CLT	0.15	0.105	0.049	0.049	0.923	0.840
0.8	0.05	62	CLT	0.11	0.141	0.150	0.311	0.960	0.910
0.8	0.10	53	CLT	0.12	0.133	0.122	0.360	0.952	0.927
0.8	0.25	40	CLT	0.15	0.112	0.107	0.107	0.928	0.869
0.8	0.30	38	CLT	0.15	0.104	0.038	0.038	0.926	0.860
0.9	0.05	50	CLT	0.11	0.145	0.161	0.336	0.961	0.914
0.9	0.10	43	CLT	0.12	0.134	0.155	0.357	0.956	0.944
0.9	0.25	32	CLT	0.15	0.112	0.095	0.095	0.930	0.874
0.9	0.30	31	CLT	0.15	0.105	0.038	0.038	0.933	0.933
1.0	0.05	41	CLT	0.11	0.141	0.153	0.319	0.964	0.926
1.0	0.10	35	CLT	0.12	0.136	0.154	0.370	0.955	0.954
1.0	0.25	26	CLT	0.15	0.113	0.099	0.099	0.928	0.863
1.0	0.30	25	CLT	0.15	0.105	0.030	0.030	0.931	0.907
1.1	0.05	34	CLT	0.11	0.143	0.150	0.317	0.964	0.928
1.1	0.10	29	CLT	0.12	0.133	0.143	0.345	0.953	0.939
1.1	0.25	22	CLT	0.15	0.113	0.097	0.097	0.931	0.891
1.1	0.30	21	CLT	0.15	0.105	0.042	0.042	0.932	0.915

Table 11: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=500, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.11	0.143	0.153	0.329	0.854	0.319
0.7	0.10	51	CLT	0.12	0.136	0.158	0.397	0.851	0.215
0.7	0.25	42	CLT	0.15	0.112	0.118	0.118	0.857	0.128
0.7	0.30	40	CLT	0.15	0.107	0.054	0.054	0.856	0.099
0.8	0.05	45	CLT	0.11	0.144	0.163	0.365	0.848	0.309
0.8	0.10	39	CLT	0.12	0.134	0.144	0.367	0.846	0.191
0.8	0.25	33	CLT	0.15	0.112	0.120	0.120	0.865	0.162
0.8	0.30	31	CLT	0.15	0.105	0.061	0.061	0.858	0.091
0.9	0.05	36	CLT	0.11	0.145	0.157	0.357	0.848	0.300
0.9	0.10	32	CLT	0.12	0.136	0.146	0.394	0.855	0.231
0.9	0.25	26	CLT	0.15	0.112	0.111	0.111	0.860	0.134
0.9	0.30	25	CLT	0.15	0.105	0.060	0.060	0.862	0.123
1.0	0.05	30	CLT	0.11	0.143	0.152	0.335	0.858	0.334
1.0	0.10	26	CLT	0.12	0.134	0.161	0.356	0.856	0.226
1.0	0.25	21	CLT	0.15	0.112	0.116	0.116	0.855	0.119
1.0	0.30	20	CLT	0.15	0.104	0.041	0.041	0.853	0.079
1.1	0.05	25	CLT	0.11	0.141	0.150	0.329	0.858	0.340
1.1	0.10	22	CLT	0.12	0.136	0.145	0.389	0.862	0.265
1.1	0.25	18	CLT	0.15	0.113	0.124	0.124	0.864	0.167
1.1	0.30	17	CLT	0.15	0.104	0.047	0.047	0.857	0.097

Table 12: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=500, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	100	CLT	0.06	0.142	0.126	0.368	0.980	0.932
0.7	0.10	84	CLT	0.07	0.141	0.164	0.453	0.983	0.931
0.7	0.25	60	CLT	0.12	0.113	0.148	0.267	0.960	0.906
0.7	0.30	56	CLT	0.13	0.105	0.142	0.216	0.952	0.899
0.8	0.05	77	CLT	0.06	0.138	0.136	0.369	0.976	0.929
0.8	0.10	65	CLT	0.07	0.134	0.144	0.410	0.983	0.926
0.8	0.25	46	CLT	0.12	0.114	0.162	0.293	0.960	0.907
0.8	0.30	43	CLT	0.13	0.104	0.147	0.196	0.952	0.881
0.9	0.05	62	CLT	0.06	0.134	0.127	0.358	0.983	0.949
0.9	0.10	51	CLT	0.07	0.127	0.149	0.380	0.982	0.920
0.9	0.25	37	CLT	0.12	0.116	0.169	0.287	0.960	0.917
0.9	0.30	34	CLT	0.13	0.105	0.169	0.229	0.952	0.879
1.0	0.05	51	CLT	0.06	0.146	0.150	0.380	0.982	0.944
1.0	0.10	42	CLT	0.07	0.135	0.155	0.430	0.983	0.920
1.0	0.25	30	CLT	0.12	0.114	0.168	0.280	0.960	0.917
1.0	0.30	28	CLT	0.13	0.106	0.167	0.210	0.955	0.900
1.1	0.05	42	CLT	0.06	0.147	0.144	0.389	0.980	0.940
1.1	0.10	35	CLT	0.07	0.138	0.157	0.431	0.978	0.919
1.1	0.25	25	CLT	0.12	0.115	0.172	0.271	0.961	0.919
1.1	0.30	23	CLT	0.13	0.108	0.182	0.241	0.954	0.895

Table 13: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=100, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.06	0.139	0.000	0.365	0.842	0.523
0.7	0.10	51	CLT	0.07	0.135	0.158	0.409	0.849	0.425
0.7	0.25	42	CLT	0.12	0.112	0.153	0.283	0.850	0.314
0.7	0.30	40	CLT	0.13	0.101	0.140	0.202	0.854	0.319
0.8	0.05	45	CLT	0.06	0.150	0.000	0.389	0.846	0.514
0.8	0.10	39	CLT	0.07	0.131	0.148	0.398	0.843	0.401
0.8	0.25	33	CLT	0.12	0.116	0.162	0.297	0.864	0.374
0.8	0.30	31	CLT	0.13	0.107	0.170	0.240	0.857	0.323
0.9	0.05	36	CLT	0.06	0.141	0.000	0.388	0.829	0.480
0.9	0.10	32	CLT	0.07	0.138	0.166	0.433	0.856	0.462
0.9	0.25	26	CLT	0.12	0.117	0.177	0.308	0.856	0.357
0.9	0.30	25	CLT	0.13	0.108	0.172	0.244	0.865	0.343
1.0	0.05	30	CLT	0.06	0.138	0.000	0.363	0.832	0.500
1.0	0.10	26	CLT	0.07	0.134	0.172	0.422	0.850	0.444
1.0	0.25	21	CLT	0.12	0.113	0.147	0.283	0.857	0.355
1.0	0.30	20	CLT	0.13	0.111	0.181	0.264	0.852	0.309
1.1	0.05	25	CLT	0.06	0.141	0.000	0.369	0.840	0.504
1.1	0.10	22	CLT	0.07	0.140	0.159	0.437	0.852	0.449
1.1	0.25	18	CLT	0.12	0.113	0.151	0.279	0.862	0.389
1.1	0.30	17	CLT	0.13	0.107	0.152	0.212	0.858	0.327

Table 14: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=100, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	120	Rom		0.157	0.130	0.380	0.909	0.905
0.7	0.10	104	Rom		0.134	0.095	0.427	0.993	0.985
0.7	0.25	67	CLT	0.10	0.113	0.158	0.325	0.975	0.929
0.7	0.30	62	CLT	0.11	0.100	0.149	0.248	0.967	0.913
0.8	0.05	92	Rom		0.129	0.113	0.333	0.908	0.906
0.8	0.10	81	Rom		0.129	0.101	0.403	0.988	0.974
0.8	0.25	52	CLT	0.10	0.112	0.157	0.329	0.976	0.932
0.8	0.30	48	CLT	0.11	0.108	0.179	0.302	0.970	0.930
0.9	0.05	74	Rom		0.151	0.117	0.362	0.918	0.915
0.9	0.10	64	Rom		0.136	0.091	0.437	0.993	0.980
0.9	0.25	41	CLT	0.10	0.115	0.165	0.324	0.974	0.925
0.9	0.30	38	CLT	0.11	0.110	0.178	0.301	0.971	0.925
1.0	0.05	60	Rom		0.148	0.131	0.367	0.922	0.921
1.0	0.10	53	Rom		0.134	0.105	0.410	0.992	0.980
1.0	0.25	34	CLT	0.10	0.112	0.151	0.322	0.974	0.927
1.0	0.30	31	CLT	0.11	0.101	0.168	0.272	0.970	0.927
1.1	0.05	50	Rom		0.145	0.126	0.360	0.920	0.916
1.1	0.10	44	Rom		0.129	0.077	0.398	0.989	0.978
1.1	0.25	28	CLT	0.10	0.110	0.148	0.315	0.979	0.949
1.1	0.30	26	CLT	0.11	0.101	0.136	0.260	0.969	0.915

Table 15: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=50, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	Rom		0.141	0.134	0.333	0.793	0.659
0.7	0.10	51	Rom		0.134	0.114	0.409	0.832	0.495
0.7	0.25	42	CLT	0.10	0.112	0.163	0.333	0.852	0.442
0.7	0.30	40	CLT	0.11	0.108	0.174	0.306	0.859	0.426
0.8	0.05	45	Rom		0.139	0.118	0.329	0.809	0.690
0.8	0.10	39	Rom		0.129	0.110	0.379	0.824	0.499
0.8	0.25	33	CLT	0.10	0.107	0.152	0.308	0.865	0.474
0.8	0.30	31	CLT	0.11	0.106	0.161	0.305	0.862	0.434
0.9	0.05	36	Rom		0.158	0.159	0.374	0.786	0.659
0.9	0.10	32	Rom		0.137	0.121	0.405	0.845	0.514
0.9	0.25	26	CLT	0.10	0.112	0.157	0.333	0.857	0.474
0.9	0.30	25	CLT	0.11	0.105	0.161	0.282	0.857	0.413
1.0	0.05	30	Rom		0.156	0.139	0.361	0.779	0.634
1.0	0.10	26	Rom		0.132	0.128	0.403	0.848	0.525
1.0	0.25	21	CLT	0.10	0.115	0.181	0.340	0.853	0.448
1.0	0.30	20	CLT	0.11	0.106	0.165	0.294	0.849	0.394
1.1	0.05	25	Rom		0.147	0.123	0.339	0.789	0.662
1.1	0.10	22	Rom		0.142	0.126	0.434	0.850	0.538
1.1	0.25	18	CLT	0.10	0.112	0.133	0.334	0.860	0.464
1.1	0.30	17	CLT	0.11	0.108	0.162	0.311	0.861	0.428

Table 16: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=50, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	132	Rom		0.147	0.135	0.263	0.654	0.654
0.7	0.10	115	Rom		0.124	0.124	0.289	0.857	0.856
0.7	0.25	91	Rom		0.124	0.104	0.398	0.993	0.981
0.7	0.30	86	Rom		0.105	0.072	0.316	0.993	0.972
0.8	0.05	102	Rom		0.145	0.141	0.247	0.648	0.648
0.8	0.10	88	Rom		0.137	0.137	0.332	0.881	0.880
0.8	0.25	70	Rom		0.114	0.096	0.366	0.993	0.982
0.8	0.30	66	Rom		0.109	0.074	0.339	0.995	0.974
0.9	0.05	81	Rom		0.144	0.129	0.238	0.638	0.638
0.9	0.10	70	Rom		0.125	0.116	0.310	0.881	0.881
0.9	0.25	56	Rom		0.109	0.078	0.342	0.992	0.977
0.9	0.30	53	Rom		0.109	0.086	0.327	0.995	0.975
1.0	0.05	66	Rom		0.148	0.136	0.274	0.661	0.661
1.0	0.10	58	Rom		0.144	0.125	0.330	0.862	0.861
1.0	0.25	46	Rom		0.106	0.088	0.327	0.992	0.979
1.0	0.30	43	Rom		0.109	0.072	0.326	0.991	0.969
1.1	0.05	55	Rom		0.142	0.132	0.248	0.638	0.638
1.1	0.10	48	Rom		0.127	0.140	0.317	0.894	0.894
1.1	0.25	38	Rom		0.120	0.093	0.375	0.993	0.978
1.1	0.30	36	Rom		0.108	0.082	0.323	0.994	0.975

Table 17: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=20, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	Rom		0.150	0.126	0.249	0.576	0.556
0.7	0.10	51	Rom		0.146	0.142	0.325	0.758	0.666
0.7	0.25	42	Rom		0.115	0.104	0.366	0.847	0.507
0.7	0.30	40	Rom		0.106	0.095	0.328	0.843	0.450
0.8	0.05	45	Rom		0.137	0.115	0.236	0.578	0.556
0.8	0.10	39	Rom		0.121	0.109	0.283	0.741	0.656
0.8	0.25	33	Rom		0.115	0.105	0.370	0.853	0.529
0.8	0.30	31	Rom		0.110	0.112	0.355	0.847	0.436
0.9	0.05	36	Rom		0.133	0.127	0.214	0.561	0.541
0.9	0.10	32	Rom		0.135	0.128	0.308	0.748	0.655
0.9	0.25	26	Rom		0.116	0.115	0.359	0.847	0.545
0.9	0.30	25	Rom		0.100	0.089	0.296	0.848	0.481
1.0	0.05	30	Rom		0.149	0.117	0.242	0.554	0.526
1.0	0.10	26	Rom		0.122	0.123	0.263	0.749	0.654
1.0	0.25	21	Rom		0.115	0.119	0.346	0.845	0.521
1.0	0.30	20	Rom		0.109	0.094	0.342	0.849	0.459
1.1	0.05	25	Rom		0.142	0.132	0.238	0.590	0.570
1.1	0.10	22	Rom		0.133	0.126	0.301	0.757	0.671
1.1	0.25	18	Rom		0.122	0.123	0.378	0.848	0.534
1.1	0.30	17	Rom		0.099	0.085	0.308	0.852	0.481

Table 18: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=20, $\rho = 0$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	68	CLT	0.15	0.143	0.058	0.058	0.925	0.902
0.7	0.10	60	FDR		0.134	0.089	0.089	0.922	0.927
0.7	0.25	49	FDR		0.112	0.000	0.000	0.919	0.979
0.7	0.30	46	FDR		0.105	0.000	0.000	0.913	0.905
0.8	0.05	53	CLT	0.15	0.144	0.048	0.048	0.929	0.931
0.8	0.10	46	FDR		0.135	0.114	0.114	0.924	0.956
0.8	0.25	38	FDR		0.113	0.000	0.000	0.922	0.984
0.8	0.30	36	FDR		0.105	0.000	0.000	0.919	0.982
0.9	0.05	42	CLT	0.15	0.143	0.050	0.050	0.931	0.944
0.9	0.10	37	FDR		0.135	0.108	0.108	0.928	0.973
0.9	0.25	30	FDR		0.112	0.000	0.000	0.922	0.988
0.9	0.30	29	FDR		0.105	0.000	0.000	0.924	0.998
1.0	0.05	35	CLT	0.15	0.142	0.036	0.036	0.939	0.977
1.0	0.10	30	FDR		0.135	0.101	0.101	0.929	0.978
1.0	0.25	25	FDR		0.113	0.000	0.000	0.930	1.000
1.0	0.30	24	FDR		0.105	0.000	0.000	0.930	1.000
1.1	0.05	29	CLT	0.15	0.142	0.047	0.047	0.939	0.977
1.1	0.10	25	FDR		0.135	0.123	0.123	0.931	0.979
1.1	0.25	21	FDR		0.113	0.000	0.000	0.933	1.000
1.1	0.30	20	FDR		0.105	0.000	0.000	0.932	1.000

Table 19: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=10000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.15	0.142	0.052	0.052	0.871	0.151
0.7	0.10	51	FDR		0.134	0.113	0.113	0.865	0.024
0.7	0.25	42	FDR		0.113	0.000	0.000	0.868	0.006
0.7	0.30	40	FDR		0.105	0.000	0.000	0.867	0.002
0.8	0.05	45	CLT	0.15	0.142	0.045	0.045	0.870	0.152
0.8	0.10	39	FDR		0.135	0.117	0.117	0.864	0.037
0.8	0.25	33	FDR		0.112	0.000	0.000	0.878	0.036
0.8	0.30	31	FDR		0.105	0.000	0.000	0.871	0.005
0.9	0.05	36	CLT	0.15	0.143	0.045	0.045	0.875	0.187
0.9	0.10	32	FDR		0.135	0.125	0.125	0.879	0.135
0.9	0.25	26	FDR		0.113	0.000	0.000	0.877	0.023
0.9	0.30	25	FDR		0.105	0.000	0.000	0.879	0.027
1.0	0.05	30	CLT	0.15	0.141	0.045	0.045	0.886	0.331
1.0	0.10	26	FDR		0.135	0.104	0.104	0.880	0.168
1.0	0.25	21	FDR		0.112	0.000	0.000	0.875	0.020
1.0	0.30	20	FDR		0.105	0.000	0.000	0.874	0.010
1.1	0.05	25	CLT	0.15	0.142	0.041	0.041	0.889	0.353
1.1	0.10	22	FDR		0.135	0.101	0.101	0.890	0.312
1.1	0.25	18	FDR		0.112	0.000	0.000	0.889	0.167
1.1	0.30	17	FDR		0.105	0.000	0.000	0.884	0.088

Table 20: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=10000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	69	CLT	0.15	0.143	0.123	0.123	0.929	0.843
0.7	0.10	60	CLT	0.15	0.135	0.163	0.188	0.923	0.871
0.7	0.25	49	FDR		0.113	0.000	0.000	0.918	0.912
0.7	0.30	47	FDR		0.105	0.000	0.000	0.920	0.939
0.8	0.05	54	CLT	0.15	0.142	0.108	0.108	0.935	0.905
0.8	0.10	47	CLT	0.15	0.135	0.159	0.183	0.929	0.940
0.8	0.25	38	FDR		0.113	0.000	0.000	0.922	0.953
0.8	0.30	36	FDR		0.105	0.000	0.000	0.919	0.926
0.9	0.05	43	CLT	0.15	0.144	0.125	0.125	0.936	0.914
0.9	0.10	37	CLT	0.15	0.135	0.156	0.182	0.928	0.917
0.9	0.25	30	FDR		0.113	0.000	0.000	0.922	0.952
0.9	0.30	29	FDR		0.105	0.000	0.000	0.924	0.964
1.0	0.05	35	CLT	0.15	0.143	0.119	0.119	0.938	0.927
1.0	0.10	31	CLT	0.15	0.136	0.179	0.196	0.938	0.975
1.0	0.25	25	FDR		0.112	0.000	0.000	0.930	0.981
1.0	0.30	24	FDR		0.105	0.000	0.000	0.930	0.995
1.1	0.05	29	CLT	0.15	0.143	0.124	0.124	0.939	0.937
1.1	0.10	26	CLT	0.15	0.134	0.157	0.173	0.941	0.979
1.1	0.25	21	FDR		0.113	0.000	0.000	0.934	0.996
1.1	0.30	20	FDR		0.105	0.000	0.000	0.932	0.994

Table 21: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=5000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.15	0.142	0.105	0.105	0.874	0.276
0.7	0.10	51	CLT	0.15	0.135	0.170	0.198	0.864	0.116
0.7	0.25	42	FDR		0.112	0.000	0.000	0.869	0.042
0.7	0.30	40	FDR		0.105	0.000	0.000	0.868	0.028
0.8	0.05	45	CLT	0.15	0.142	0.119	0.119	0.868	0.225
0.8	0.10	39	CLT	0.15	0.135	0.162	0.191	0.864	0.100
0.8	0.25	33	FDR		0.113	0.000	0.000	0.878	0.104
0.8	0.30	31	FDR		0.105	0.000	0.000	0.872	0.043
0.9	0.05	36	CLT	0.15	0.142	0.106	0.106	0.875	0.276
0.9	0.10	32	CLT	0.15	0.135	0.159	0.198	0.878	0.237
0.9	0.25	26	FDR		0.112	0.000	0.000	0.877	0.093
0.9	0.30	25	FDR		0.105	0.000	0.000	0.878	0.079
1.0	0.05	30	CLT	0.15	0.143	0.110	0.110	0.887	0.415
1.0	0.10	26	CLT	0.15	0.135	0.158	0.185	0.880	0.251
1.0	0.25	21	FDR		0.113	0.001	0.001	0.876	0.080
1.0	0.30	20	FDR		0.105	0.000	0.000	0.875	0.061
1.1	0.05	25	CLT	0.15	0.141	0.114	0.114	0.888	0.411
1.1	0.10	22	CLT	0.15	0.135	0.162	0.191	0.889	0.369
1.1	0.25	18	FDR		0.113	0.001	0.001	0.889	0.270
1.1	0.30	17	FDR		0.105	0.000	0.000	0.884	0.153

Table 22: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=5000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	72	CLT	0.14	0.142	0.168	0.218	0.940	0.859
0.7	0.10	63	CLT	0.14	0.135	0.164	0.294	0.935	0.876
0.7	0.25	50	FDR		0.112	0.011	0.011	0.925	0.888
0.7	0.30	48	FDR		0.105	0.001	0.001	0.923	0.892
0.8	0.05	56	CLT	0.14	0.144	0.181	0.222	0.944	0.878
0.8	0.10	48	CLT	0.14	0.135	0.157	0.272	0.934	0.891
0.8	0.25	38	FDR		0.112	0.016	0.016	0.923	0.876
0.8	0.30	37	FDR		0.106	0.002	0.002	0.926	0.917
0.9	0.05	44	CLT	0.14	0.141	0.149	0.199	0.942	0.877
0.9	0.10	39	CLT	0.14	0.135	0.153	0.290	0.942	0.925
0.9	0.25	31	FDR		0.113	0.015	0.015	0.931	0.940
0.9	0.30	29	FDR		0.105	0.002	0.002	0.924	0.897
1.0	0.05	36	CLT	0.14	0.143	0.170	0.213	0.944	0.889
1.0	0.10	32	CLT	0.14	0.133	0.165	0.266	0.945	0.942
1.0	0.25	25	FDR		0.112	0.018	0.018	0.929	0.922
1.0	0.30	24	FDR		0.105	0.001	0.001	0.930	0.959
1.1	0.05	31	CLT	0.14	0.141	0.146	0.181	0.954	0.937
1.1	0.10	27	CLT	0.14	0.135	0.164	0.280	0.950	0.959
1.1	0.25	21	FDR		0.113	0.018	0.018	0.934	0.953
1.1	0.30	20	FDR		0.105	0.001	0.001	0.932	0.963

Table 23: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=2000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.14	0.142	0.175	0.235	0.873	0.362
0.7	0.10	51	CLT	0.14	0.135	0.167	0.302	0.862	0.230
0.7	0.25	42	FDR		0.112	0.013	0.013	0.868	0.144
0.7	0.30	40	FDR		0.105	0.007	0.007	0.867	0.098
0.8	0.05	45	CLT	0.14	0.144	0.174	0.241	0.869	0.352
0.8	0.10	39	CLT	0.14	0.136	0.182	0.312	0.863	0.224
0.8	0.25	33	FDR		0.112	0.016	0.016	0.877	0.187
0.8	0.30	31	FDR		0.105	0.004	0.004	0.871	0.132
0.9	0.05	36	CLT	0.14	0.142	0.151	0.203	0.870	0.359
0.9	0.10	32	CLT	0.14	0.133	0.141	0.254	0.878	0.346
0.9	0.25	26	FDR		0.113	0.012	0.012	0.876	0.205
0.9	0.30	25	FDR		0.105	0.001	0.001	0.878	0.206
1.0	0.05	30	CLT	0.14	0.141	0.146	0.197	0.887	0.490
1.0	0.10	26	CLT	0.14	0.134	0.140	0.247	0.879	0.338
1.0	0.25	21	FDR		0.113	0.026	0.026	0.876	0.208
1.0	0.30	20	FDR		0.104	0.002	0.002	0.874	0.159
1.1	0.05	25	CLT	0.14	0.143	0.155	0.212	0.892	0.511
1.1	0.10	22	CLT	0.14	0.135	0.164	0.276	0.888	0.436
1.1	0.25	18	FDR		0.113	0.018	0.018	0.888	0.344
1.1	0.30	17	FDR		0.104	0.004	0.004	0.886	0.294

Table 24: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=2000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	76	CLT	0.13	0.139	0.156	0.256	0.950	0.857
0.7	0.10	65	CLT	0.13	0.135	0.156	0.332	0.944	0.876
0.7	0.25	51	CLT	0.15	0.113	0.066	0.066	0.929	0.856
0.7	0.30	48	CLT	0.15	0.105	0.016	0.016	0.926	0.854
0.8	0.05	58	CLT	0.13	0.145	0.165	0.286	0.951	0.875
0.8	0.10	50	CLT	0.13	0.136	0.181	0.336	0.944	0.889
0.8	0.25	39	CLT	0.15	0.111	0.047	0.047	0.928	0.851
0.8	0.30	37	CLT	0.15	0.105	0.016	0.016	0.927	0.856
0.9	0.05	47	CLT	0.13	0.143	0.172	0.295	0.955	0.888
0.9	0.10	40	CLT	0.13	0.137	0.171	0.354	0.947	0.894
0.9	0.25	31	CLT	0.15	0.112	0.057	0.057	0.930	0.867
0.9	0.30	30	CLT	0.15	0.105	0.014	0.014	0.934	0.916
1.0	0.05	38	CLT	0.13	0.143	0.162	0.287	0.958	0.905
1.0	0.10	33	CLT	0.13	0.133	0.151	0.308	0.951	0.920
1.0	0.25	26	CLT	0.15	0.113	0.066	0.066	0.939	0.931
1.0	0.30	24	CLT	0.15	0.105	0.021	0.021	0.929	0.878
1.1	0.05	32	CLT	0.13	0.147	0.188	0.302	0.960	0.915
1.1	0.10	28	CLT	0.13	0.135	0.177	0.349	0.957	0.953
1.1	0.25	22	CLT	0.15	0.113	0.046	0.046	0.944	0.942
1.1	0.30	21	CLT	0.15	0.105	0.025	0.025	0.941	0.961

Table 25: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=1000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.13	0.144	0.150	0.283	0.869	0.446
0.7	0.10	51	CLT	0.13	0.135	0.171	0.347	0.859	0.293
0.7	0.25	42	CLT	0.15	0.113	0.063	0.063	0.870	0.239
0.7	0.30	40	CLT	0.15	0.104	0.022	0.022	0.866	0.187
0.8	0.05	45	CLT	0.13	0.141	0.164	0.273	0.868	0.434
0.8	0.10	39	CLT	0.13	0.134	0.179	0.342	0.862	0.321
0.8	0.25	33	CLT	0.15	0.112	0.054	0.054	0.876	0.321
0.8	0.30	31	CLT	0.15	0.103	0.019	0.019	0.870	0.218
0.9	0.05	36	CLT	0.13	0.143	0.156	0.285	0.871	0.466
0.9	0.10	32	CLT	0.13	0.136	0.171	0.345	0.879	0.434
0.9	0.25	26	CLT	0.15	0.114	0.072	0.072	0.875	0.307
0.9	0.30	25	CLT	0.15	0.104	0.023	0.023	0.876	0.278
1.0	0.05	30	CLT	0.13	0.140	0.153	0.278	0.886	0.537
1.0	0.10	26	CLT	0.13	0.134	0.163	0.333	0.876	0.405
1.0	0.25	21	CLT	0.15	0.112	0.063	0.063	0.873	0.280
1.0	0.30	20	CLT	0.15	0.106	0.025	0.025	0.873	0.241
1.1	0.05	25	CLT	0.13	0.142	0.149	0.274	0.884	0.513
1.1	0.10	22	CLT	0.13	0.138	0.172	0.361	0.886	0.471
1.1	0.25	18	CLT	0.15	0.112	0.056	0.056	0.887	0.406
1.1	0.30	17	CLT	0.15	0.105	0.024	0.024	0.883	0.348

Table 26: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=1000, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	81	CLT	0.11	0.145	0.162	0.320	0.967	0.920
0.7	0.10	69	CLT	0.12	0.133	0.157	0.353	0.954	0.889
0.7	0.25	52	CLT	0.15	0.111	0.108	0.108	0.932	0.817
0.7	0.30	49	CLT	0.15	0.104	0.065	0.065	0.931	0.834
0.8	0.05	62	CLT	0.11	0.142	0.168	0.325	0.965	0.915
0.8	0.10	53	CLT	0.12	0.133	0.153	0.363	0.956	0.894
0.8	0.25	40	CLT	0.15	0.113	0.135	0.135	0.935	0.854
0.8	0.30	38	CLT	0.15	0.104	0.068	0.068	0.932	0.839
0.9	0.05	50	CLT	0.11	0.145	0.164	0.335	0.969	0.931
0.9	0.10	43	CLT	0.12	0.134	0.163	0.378	0.962	0.929
0.9	0.25	32	CLT	0.15	0.113	0.130	0.130	0.937	0.866
0.9	0.30	31	CLT	0.15	0.105	0.061	0.061	0.940	0.890
1.0	0.05	41	CLT	0.11	0.143	0.166	0.319	0.972	0.939
1.0	0.10	35	CLT	0.12	0.134	0.187	0.357	0.961	0.917
1.0	0.25	26	CLT	0.15	0.114	0.130	0.130	0.940	0.886
1.0	0.30	25	CLT	0.15	0.105	0.066	0.066	0.940	0.875
1.1	0.05	34	CLT	0.11	0.144	0.165	0.337	0.971	0.929
1.1	0.10	29	CLT	0.12	0.133	0.156	0.358	0.962	0.919
1.1	0.25	22	CLT	0.15	0.112	0.117	0.117	0.944	0.901
1.1	0.30	21	CLT	0.15	0.104	0.062	0.062	0.944	0.904

Table 27: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=500, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.11	0.144	0.165	0.343	0.864	0.461
0.7	0.10	51	CLT	0.12	0.136	0.174	0.394	0.859	0.403
0.7	0.25	42	CLT	0.15	0.111	0.127	0.127	0.869	0.348
0.7	0.30	40	CLT	0.15	0.105	0.062	0.062	0.867	0.292
0.8	0.05	45	CLT	0.11	0.140	0.140	0.327	0.864	0.469
0.8	0.10	39	CLT	0.12	0.137	0.167	0.380	0.859	0.391
0.8	0.25	33	CLT	0.15	0.114	0.137	0.137	0.875	0.383
0.8	0.30	31	CLT	0.15	0.104	0.069	0.069	0.870	0.313
0.9	0.05	36	CLT	0.11	0.143	0.170	0.332	0.873	0.498
0.9	0.10	32	CLT	0.12	0.134	0.158	0.375	0.878	0.482
0.9	0.25	26	CLT	0.15	0.111	0.139	0.139	0.876	0.360
0.9	0.30	25	CLT	0.15	0.104	0.065	0.065	0.876	0.341
1.0	0.05	30	CLT	0.11	0.140	0.160	0.314	0.877	0.504
1.0	0.10	26	CLT	0.12	0.137	0.152	0.382	0.880	0.500
1.0	0.25	21	CLT	0.15	0.112	0.114	0.114	0.878	0.388
1.0	0.30	20	CLT	0.15	0.103	0.063	0.063	0.875	0.360
1.1	0.05	25	CLT	0.11	0.144	0.165	0.346	0.881	0.541
1.1	0.10	22	CLT	0.12	0.136	0.182	0.378	0.884	0.521
1.1	0.25	18	CLT	0.15	0.113	0.125	0.125	0.891	0.500
1.1	0.30	17	CLT	0.15	0.105	0.072	0.072	0.882	0.405

Table 28: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=500, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	100	CLT	0.06	0.145	0.167	0.376	0.984	0.960
0.7	0.10	84	CLT	0.07	0.127	0.130	0.371	0.985	0.932
0.7	0.25	60	CLT	0.12	0.113	0.175	0.286	0.960	0.883
0.7	0.30	56	CLT	0.13	0.103	0.161	0.210	0.958	0.876
0.8	0.05	77	CLT	0.06	0.137	0.150	0.350	0.982	0.951
0.8	0.10	65	CLT	0.07	0.134	0.164	0.403	0.983	0.926
0.8	0.25	46	CLT	0.12	0.111	0.159	0.276	0.961	0.891
0.8	0.30	43	CLT	0.13	0.107	0.181	0.228	0.956	0.884
0.9	0.05	62	CLT	0.06	0.148	0.169	0.377	0.981	0.953
0.9	0.10	51	CLT	0.07	0.142	0.172	0.439	0.980	0.929
0.9	0.25	37	CLT	0.12	0.112	0.162	0.263	0.965	0.918
0.9	0.30	34	CLT	0.13	0.106	0.172	0.219	0.955	0.871
1.0	0.05	51	CLT	0.06	0.146	0.155	0.381	0.987	0.960
1.0	0.10	42	CLT	0.07	0.139	0.166	0.414	0.984	0.947
1.0	0.25	30	CLT	0.12	0.114	0.169	0.280	0.967	0.924
1.0	0.30	28	CLT	0.13	0.105	0.171	0.216	0.958	0.879
1.1	0.05	42	CLT	0.06	0.149	0.155	0.384	0.988	0.966
1.1	0.10	35	CLT	0.07	0.132	0.145	0.388	0.983	0.923
1.1	0.25	25	CLT	0.12	0.109	0.153	0.260	0.966	0.921
1.1	0.30	23	CLT	0.13	0.103	0.158	0.212	0.960	0.901

Table 29: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=100, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.06	0.140	0.000	0.356	0.856	0.553
0.7	0.10	51	CLT	0.07	0.132	0.156	0.412	0.856	0.491
0.7	0.25	42	CLT	0.12	0.113	0.178	0.294	0.864	0.470
0.7	0.30	40	CLT	0.13	0.105	0.180	0.236	0.862	0.468
0.8	0.05	45	CLT	0.06	0.140	0.000	0.358	0.862	0.575
0.8	0.10	39	CLT	0.07	0.137	0.164	0.413	0.855	0.496
0.8	0.25	33	CLT	0.12	0.118	0.181	0.302	0.871	0.480
0.8	0.30	31	CLT	0.13	0.106	0.175	0.240	0.872	0.487
0.9	0.05	36	CLT	0.06	0.135	0.000	0.348	0.869	0.615
0.9	0.10	32	CLT	0.07	0.132	0.156	0.403	0.875	0.549
0.9	0.25	26	CLT	0.12	0.112	0.165	0.279	0.869	0.466
0.9	0.30	25	CLT	0.13	0.105	0.148	0.228	0.875	0.486
1.0	0.05	30	CLT	0.06	0.140	0.000	0.351	0.880	0.632
1.0	0.10	26	CLT	0.07	0.132	0.148	0.398	0.882	0.559
1.0	0.25	21	CLT	0.12	0.107	0.135	0.245	0.870	0.497
1.0	0.30	20	CLT	0.13	0.102	0.167	0.217	0.864	0.457
1.1	0.05	25	CLT	0.06	0.144	0.000	0.359	0.875	0.624
1.1	0.10	22	CLT	0.07	0.136	0.166	0.413	0.880	0.565
1.1	0.25	18	CLT	0.12	0.111	0.157	0.284	0.889	0.550
1.1	0.30	17	CLT	0.13	0.102	0.164	0.209	0.883	0.516

Table 30: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=100, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	120	Rom		0.138	0.121	0.339	0.928	0.925
0.7	0.10	104	Rom		0.134	0.095	0.407	0.990	0.977
0.7	0.25	67	CLT	0.10	0.105	0.135	0.288	0.978	0.937
0.7	0.30	62	CLT	0.11	0.103	0.173	0.277	0.970	0.918
0.8	0.05	92	Rom		0.139	0.134	0.344	0.927	0.925
0.8	0.10	81	Rom		0.134	0.104	0.397	0.988	0.971
0.8	0.25	52	CLT	0.10	0.113	0.169	0.313	0.976	0.920
0.8	0.30	48	CLT	0.11	0.100	0.154	0.249	0.973	0.917
0.9	0.05	74	Rom		0.133	0.128	0.319	0.940	0.938
0.9	0.10	64	Rom		0.136	0.094	0.407	0.989	0.982
0.9	0.25	41	CLT	0.10	0.110	0.174	0.299	0.980	0.941
0.9	0.30	38	CLT	0.11	0.103	0.165	0.267	0.973	0.925
1.0	0.05	60	Rom		0.145	0.135	0.348	0.918	0.917
1.0	0.10	53	Rom		0.128	0.090	0.396	0.996	0.983
1.0	0.25	34	CLT	0.10	0.106	0.130	0.276	0.981	0.941
1.0	0.30	31	CLT	0.11	0.109	0.183	0.304	0.973	0.918
1.1	0.05	50	Rom		0.133	0.110	0.325	0.924	0.923
1.1	0.10	44	Rom		0.137	0.117	0.404	0.990	0.983
1.1	0.25	28	CLT	0.10	0.114	0.174	0.323	0.978	0.932
1.1	0.30	26	CLT	0.11	0.104	0.153	0.275	0.972	0.921

Table 31: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=50, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	Rom		0.138	0.132	0.312	0.791	0.676
0.7	0.10	51	Rom		0.128	0.120	0.386	0.847	0.558
0.7	0.25	42	CLT	0.10	0.113	0.153	0.325	0.859	0.499
0.7	0.30	40	CLT	0.11	0.106	0.163	0.297	0.868	0.502
0.8	0.05	45	Rom		0.139	0.120	0.323	0.796	0.674
0.8	0.10	39	Rom		0.126	0.106	0.378	0.872	0.601
0.8	0.25	33	CLT	0.10	0.108	0.146	0.304	0.865	0.523
0.8	0.30	31	CLT	0.11	0.108	0.187	0.311	0.865	0.489
0.9	0.05	36	Rom		0.137	0.121	0.315	0.804	0.683
0.9	0.10	32	Rom		0.131	0.123	0.386	0.867	0.599
0.9	0.25	26	CLT	0.10	0.112	0.140	0.312	0.867	0.509
0.9	0.30	25	CLT	0.11	0.101	0.153	0.283	0.868	0.506
1.0	0.05	30	Rom		0.156	0.132	0.351	0.814	0.708
1.0	0.10	26	Rom		0.141	0.099	0.418	0.863	0.595
1.0	0.25	21	CLT	0.10	0.111	0.156	0.308	0.873	0.539
1.0	0.30	20	CLT	0.11	0.110	0.174	0.294	0.865	0.501
1.1	0.05	25	Rom		0.128	0.122	0.299	0.809	0.704
1.1	0.10	22	Rom		0.128	0.104	0.392	0.871	0.614
1.1	0.25	18	CLT	0.10	0.110	0.152	0.313	0.893	0.609
1.1	0.30	17	CLT	0.11	0.106	0.164	0.284	0.876	0.524

Table 32: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=50, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	132	Rom		0.149	0.144	0.251	0.612	0.612
0.7	0.10	115	Rom		0.151	0.152	0.337	0.876	0.876
0.7	0.25	91	Rom		0.112	0.090	0.360	0.994	0.982
0.7	0.30	86	Rom		0.101	0.081	0.305	0.995	0.973
0.8	0.05	102	Rom		0.142	0.122	0.247	0.652	0.652
0.8	0.10	88	Rom		0.133	0.126	0.303	0.876	0.874
0.8	0.25	70	Rom		0.116	0.089	0.358	0.997	0.985
0.8	0.30	66	Rom		0.105	0.071	0.301	0.995	0.976
0.9	0.05	81	Rom		0.140	0.130	0.236	0.636	0.636
0.9	0.10	70	Rom		0.137	0.120	0.322	0.873	0.871
0.9	0.25	56	Rom		0.107	0.087	0.337	0.990	0.980
0.9	0.30	53	Rom		0.105	0.067	0.311	0.995	0.979
1.0	0.05	66	Rom		0.148	0.150	0.250	0.633	0.633
1.0	0.10	58	Rom		0.145	0.161	0.335	0.869	0.867
1.0	0.25	46	Rom		0.116	0.096	0.356	0.995	0.984
1.0	0.30	43	Rom		0.106	0.078	0.325	0.997	0.982
1.1	0.05	55	Rom		0.139	0.131	0.231	0.650	0.650
1.1	0.10	48	Rom		0.139	0.131	0.319	0.868	0.867
1.1	0.25	38	Rom		0.110	0.095	0.339	0.993	0.981
1.1	0.30	36	Rom		0.107	0.077	0.309	0.994	0.974

Table 33: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=20, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	Rom		0.146	0.132	0.239	0.568	0.555
0.7	0.10	51	Rom		0.135	0.116	0.299	0.769	0.683
0.7	0.25	42	Rom		0.117	0.118	0.339	0.846	0.555
0.7	0.30	40	Rom		0.112	0.109	0.335	0.864	0.536
0.8	0.05	45	Rom		0.136	0.129	0.222	0.554	0.538
0.8	0.10	39	Rom		0.135	0.138	0.289	0.745	0.649
0.8	0.25	33	Rom		0.108	0.098	0.328	0.869	0.603
0.8	0.30	31	Rom		0.108	0.095	0.335	0.856	0.535
0.9	0.05	36	Rom		0.148	0.147	0.245	0.579	0.559
0.9	0.10	32	Rom		0.133	0.114	0.292	0.769	0.693
0.9	0.25	26	Rom		0.114	0.107	0.362	0.876	0.608
0.9	0.30	25	Rom		0.101	0.093	0.312	0.874	0.569
1.0	0.05	30	Rom		0.133	0.124	0.215	0.571	0.555
1.0	0.10	26	Rom		0.142	0.151	0.321	0.794	0.711
1.0	0.25	21	Rom		0.108	0.105	0.321	0.861	0.573
1.0	0.30	20	Rom		0.112	0.110	0.344	0.858	0.521
1.1	0.05	25	Rom		0.143	0.147	0.237	0.585	0.572
1.1	0.10	22	Rom		0.132	0.120	0.301	0.793	0.722
1.1	0.25	18	Rom		0.114	0.103	0.345	0.881	0.608
1.1	0.30	17	Rom		0.107	0.088	0.321	0.880	0.581

Table 34: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=20, $\rho = 0.1$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	68	CLT	0.15	0.142	0.067	0.067	0.925	0.844
0.7	0.10	60	FDR		0.135	0.160	0.160	0.923	0.882
0.7	0.25	49	FDR		0.113	0.000	0.000	0.918	0.918
0.7	0.30	46	FDR		0.105	0.000	0.000	0.913	0.843
0.8	0.05	53	CLT	0.15	0.143	0.078	0.078	0.929	0.869
0.8	0.10	46	FDR		0.135	0.160	0.160	0.923	0.889
0.8	0.25	38	FDR		0.112	0.000	0.000	0.923	0.966
0.8	0.30	36	FDR		0.105	0.000	0.000	0.919	0.935
0.9	0.05	42	CLT	0.15	0.142	0.069	0.069	0.930	0.888
0.9	0.10	37	FDR		0.135	0.157	0.157	0.928	0.932
0.9	0.25	30	FDR		0.113	0.000	0.000	0.922	0.956
0.9	0.30	29	FDR		0.105	0.000	0.000	0.925	0.986
1.0	0.05	35	CLT	0.15	0.143	0.073	0.073	0.938	0.942
1.0	0.10	30	FDR		0.135	0.166	0.166	0.930	0.933
1.0	0.25	25	FDR		0.113	0.000	0.000	0.929	0.992
1.0	0.30	24	FDR		0.105	0.000	0.000	0.929	0.999
1.1	0.05	29	CLT	0.15	0.141	0.070	0.070	0.939	0.922
1.1	0.10	25	FDR		0.135	0.162	0.162	0.931	0.954
1.1	0.25	21	FDR		0.112	0.000	0.000	0.933	0.994
1.1	0.30	20	FDR		0.105	0.000	0.000	0.931	0.999

Table 35: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=10000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.15	0.142	0.080	0.080	0.871	0.227
0.7	0.10	51	FDR		0.136	0.163	0.163	0.866	0.103
0.7	0.25	42	FDR		0.112	0.000	0.000	0.868	0.017
0.7	0.30	40	FDR		0.105	0.000	0.000	0.867	0.016
0.8	0.05	45	CLT	0.15	0.142	0.074	0.074	0.869	0.209
0.8	0.10	39	FDR		0.134	0.137	0.137	0.864	0.103
0.8	0.25	33	FDR		0.112	0.000	0.000	0.879	0.097
0.8	0.30	31	FDR		0.105	0.000	0.000	0.871	0.022
0.9	0.05	36	CLT	0.15	0.142	0.072	0.072	0.875	0.260
0.9	0.10	32	FDR		0.135	0.148	0.148	0.880	0.210
0.9	0.25	26	FDR		0.112	0.000	0.000	0.876	0.073
0.9	0.30	25	FDR		0.105	0.000	0.000	0.879	0.070
1.0	0.05	30	CLT	0.15	0.143	0.067	0.067	0.885	0.363
1.0	0.10	26	FDR		0.135	0.159	0.159	0.883	0.260
1.0	0.25	21	FDR		0.112	0.000	0.000	0.875	0.068
1.0	0.30	20	FDR		0.105	0.000	0.000	0.874	0.038
1.1	0.05	25	CLT	0.15	0.143	0.074	0.074	0.890	0.421
1.1	0.10	22	FDR		0.135	0.153	0.153	0.889	0.346
1.1	0.25	18	FDR		0.112	0.000	0.000	0.889	0.247
1.1	0.30	17	FDR		0.104	0.000	0.000	0.884	0.140

Table 36: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=10000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	69	CLT	0.15	0.143	0.136	0.136	0.928	0.802
0.7	0.10	60	CLT	0.15	0.134	0.192	0.207	0.923	0.826
0.7	0.25	49	FDR		0.112	0.003	0.003	0.918	0.845
0.7	0.30	47	FDR		0.105	0.000	0.000	0.919	0.873
0.8	0.05	54	CLT	0.15	0.144	0.150	0.150	0.934	0.856
0.8	0.10	47	CLT	0.15	0.136	0.229	0.246	0.930	0.875
0.8	0.25	38	FDR		0.112	0.007	0.007	0.921	0.889
0.8	0.30	36	FDR		0.105	0.000	0.000	0.920	0.892
0.9	0.05	43	CLT	0.15	0.142	0.147	0.147	0.936	0.870
0.9	0.10	37	CLT	0.15	0.135	0.193	0.214	0.927	0.859
0.9	0.25	30	FDR		0.112	0.004	0.004	0.921	0.891
0.9	0.30	29	FDR		0.105	0.000	0.000	0.925	0.948
1.0	0.05	35	CLT	0.15	0.142	0.150	0.150	0.938	0.866
1.0	0.10	31	CLT	0.15	0.136	0.211	0.243	0.938	0.933
1.0	0.25	25	FDR		0.112	0.002	0.002	0.929	0.944
1.0	0.30	24	FDR		0.105	0.001	0.001	0.929	0.963
1.1	0.05	29	CLT	0.15	0.141	0.148	0.148	0.939	0.885
1.1	0.10	26	CLT	0.15	0.135	0.223	0.246	0.941	0.958
1.1	0.25	21	FDR		0.113	0.001	0.001	0.933	0.979
1.1	0.30	20	FDR		0.105	0.000	0.000	0.932	0.978

Table 37: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=5000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.15	0.144	0.162	0.162	0.872	0.340
0.7	0.10	51	CLT	0.15	0.137	0.223	0.261	0.866	0.189
0.7	0.25	42	FDR		0.113	0.007	0.007	0.867	0.078
0.7	0.30	40	FDR		0.105	0.000	0.000	0.867	0.068
0.8	0.05	45	CLT	0.15	0.143	0.137	0.137	0.866	0.299
0.8	0.10	39	CLT	0.15	0.136	0.212	0.231	0.864	0.187
0.8	0.25	33	FDR		0.113	0.007	0.007	0.878	0.194
0.8	0.30	31	FDR		0.105	0.000	0.000	0.871	0.084
0.9	0.05	36	CLT	0.15	0.142	0.137	0.137	0.870	0.321
0.9	0.10	32	CLT	0.15	0.136	0.222	0.245	0.880	0.320
0.9	0.25	26	FDR		0.113	0.004	0.004	0.876	0.155
0.9	0.30	25	FDR		0.105	0.000	0.000	0.879	0.152
1.0	0.05	30	CLT	0.15	0.142	0.133	0.133	0.885	0.406
1.0	0.10	26	CLT	0.15	0.134	0.185	0.216	0.880	0.329
1.0	0.25	21	FDR		0.112	0.003	0.003	0.875	0.149
1.0	0.30	20	FDR		0.105	0.000	0.000	0.874	0.115
1.1	0.05	25	CLT	0.15	0.142	0.134	0.134	0.887	0.461
1.1	0.10	22	CLT	0.15	0.135	0.208	0.237	0.889	0.412
1.1	0.25	18	FDR		0.113	0.004	0.004	0.888	0.326
1.1	0.30	17	FDR		0.105	0.000	0.000	0.883	0.210

Table 38: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=5000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	72	CLT	0.14	0.142	0.187	0.239	0.940	0.802
0.7	0.10	63	CLT	0.14	0.136	0.208	0.317	0.936	0.841
0.7	0.25	50	FDR		0.112	0.032	0.032	0.926	0.835
0.7	0.30	48	FDR		0.105	0.012	0.012	0.925	0.840
0.8	0.05	56	CLT	0.14	0.144	0.205	0.249	0.942	0.845
0.8	0.10	48	CLT	0.14	0.136	0.201	0.306	0.934	0.839
0.8	0.25	38	FDR		0.113	0.031	0.031	0.923	0.802
0.8	0.30	37	FDR		0.105	0.010	0.010	0.926	0.853
0.9	0.05	44	CLT	0.14	0.143	0.178	0.212	0.942	0.840
0.9	0.10	39	CLT	0.14	0.135	0.192	0.296	0.943	0.898
0.9	0.25	31	FDR		0.113	0.034	0.034	0.930	0.869
0.9	0.30	29	FDR		0.105	0.014	0.014	0.923	0.841
1.0	0.05	36	CLT	0.14	0.142	0.197	0.243	0.944	0.851
1.0	0.10	32	CLT	0.14	0.136	0.204	0.315	0.944	0.896
1.0	0.25	25	FDR		0.112	0.039	0.039	0.929	0.863
1.0	0.30	24	FDR		0.105	0.012	0.012	0.930	0.892
1.1	0.05	31	CLT	0.14	0.141	0.191	0.223	0.955	0.913
1.1	0.10	27	CLT	0.14	0.135	0.210	0.300	0.949	0.921
1.1	0.25	21	FDR		0.112	0.037	0.037	0.933	0.903
1.1	0.30	20	FDR		0.105	0.011	0.011	0.932	0.917

Table 39: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=2000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.14	0.142	0.184	0.234	0.864	0.406
0.7	0.10	51	CLT	0.14	0.136	0.218	0.323	0.862	0.318
0.7	0.25	42	FDR		0.113	0.041	0.041	0.868	0.216
0.7	0.30	40	FDR		0.106	0.017	0.017	0.865	0.177
0.8	0.05	45	CLT	0.14	0.143	0.187	0.244	0.866	0.423
0.8	0.10	39	CLT	0.14	0.136	0.209	0.337	0.859	0.296
0.8	0.25	33	FDR		0.111	0.033	0.033	0.877	0.272
0.8	0.30	31	FDR		0.105	0.012	0.012	0.872	0.215
0.9	0.05	36	CLT	0.14	0.144	0.213	0.260	0.873	0.479
0.9	0.10	32	CLT	0.14	0.137	0.215	0.327	0.877	0.405
0.9	0.25	26	FDR		0.113	0.060	0.060	0.875	0.282
0.9	0.30	25	FDR		0.104	0.014	0.014	0.876	0.256
1.0	0.05	30	CLT	0.14	0.140	0.176	0.237	0.882	0.526
1.0	0.10	26	CLT	0.14	0.136	0.219	0.307	0.880	0.416
1.0	0.25	21	FDR		0.112	0.039	0.039	0.875	0.283
1.0	0.30	20	FDR		0.105	0.014	0.014	0.875	0.247
1.1	0.05	25	CLT	0.14	0.141	0.188	0.240	0.882	0.488
1.1	0.10	22	CLT	0.14	0.135	0.190	0.298	0.888	0.472
1.1	0.25	18	FDR		0.112	0.041	0.041	0.890	0.414
1.1	0.30	17	FDR		0.104	0.015	0.015	0.885	0.340

Table 40: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=2000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	76	CLT	0.13	0.145	0.201	0.316	0.946	0.836
0.7	0.10	65	CLT	0.13	0.135	0.201	0.343	0.943	0.837
0.7	0.25	51	CLT	0.15	0.113	0.105	0.105	0.926	0.786
0.7	0.30	48	CLT	0.15	0.105	0.054	0.054	0.924	0.786
0.8	0.05	58	CLT	0.13	0.141	0.179	0.286	0.951	0.852
0.8	0.10	50	CLT	0.13	0.134	0.194	0.347	0.942	0.846
0.8	0.25	39	CLT	0.15	0.113	0.103	0.103	0.929	0.816
0.8	0.30	37	CLT	0.15	0.106	0.053	0.053	0.926	0.805
0.9	0.05	47	CLT	0.13	0.143	0.195	0.282	0.956	0.875
0.9	0.10	40	CLT	0.13	0.133	0.189	0.331	0.945	0.856
0.9	0.25	31	CLT	0.15	0.114	0.104	0.104	0.930	0.828
0.9	0.30	30	CLT	0.15	0.105	0.049	0.049	0.933	0.851
1.0	0.05	38	CLT	0.13	0.139	0.163	0.266	0.959	0.895
1.0	0.10	33	CLT	0.13	0.136	0.212	0.354	0.952	0.893
1.0	0.25	26	CLT	0.15	0.111	0.097	0.097	0.939	0.886
1.0	0.30	24	CLT	0.15	0.104	0.046	0.046	0.929	0.828
1.1	0.05	32	CLT	0.13	0.142	0.186	0.274	0.959	0.889
1.1	0.10	28	CLT	0.13	0.135	0.219	0.347	0.956	0.894
1.1	0.25	22	CLT	0.15	0.112	0.102	0.102	0.943	0.908
1.1	0.30	21	CLT	0.15	0.105	0.051	0.051	0.941	0.916

Table 41: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=1000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.13	0.141	0.182	0.283	0.865	0.485
0.7	0.10	51	CLT	0.13	0.135	0.195	0.352	0.861	0.391
0.7	0.25	42	CLT	0.15	0.112	0.115	0.115	0.866	0.310
0.7	0.30	40	CLT	0.15	0.104	0.052	0.052	0.867	0.280
0.8	0.05	45	CLT	0.13	0.142	0.183	0.304	0.859	0.465
0.8	0.10	39	CLT	0.13	0.132	0.171	0.331	0.864	0.413
0.8	0.25	33	CLT	0.15	0.112	0.110	0.110	0.877	0.387
0.8	0.30	31	CLT	0.15	0.104	0.056	0.056	0.870	0.328
0.9	0.05	36	CLT	0.13	0.143	0.197	0.306	0.867	0.504
0.9	0.10	32	CLT	0.13	0.136	0.197	0.345	0.873	0.456
0.9	0.25	26	CLT	0.15	0.112	0.094	0.094	0.876	0.374
0.9	0.30	25	CLT	0.15	0.105	0.057	0.057	0.877	0.368
1.0	0.05	30	CLT	0.13	0.141	0.169	0.267	0.879	0.566
1.0	0.10	26	CLT	0.13	0.133	0.191	0.341	0.884	0.498
1.0	0.25	21	CLT	0.15	0.115	0.114	0.114	0.876	0.390
1.0	0.30	20	CLT	0.15	0.106	0.066	0.066	0.874	0.314
1.1	0.05	25	CLT	0.13	0.142	0.173	0.292	0.886	0.577
1.1	0.10	22	CLT	0.13	0.134	0.177	0.323	0.885	0.505
1.1	0.25	18	CLT	0.15	0.112	0.100	0.100	0.889	0.474
1.1	0.30	17	CLT	0.15	0.106	0.065	0.065	0.883	0.384

Table 42: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=1000, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	81	CLT	0.11	0.145	0.186	0.338	0.966	0.905
0.7	0.10	69	CLT	0.12	0.133	0.197	0.369	0.956	0.877
0.7	0.25	52	CLT	0.15	0.113	0.169	0.169	0.932	0.787
0.7	0.30	49	CLT	0.15	0.105	0.107	0.107	0.927	0.757
0.8	0.05	62	CLT	0.11	0.141	0.161	0.326	0.966	0.909
0.8	0.10	53	CLT	0.12	0.134	0.194	0.360	0.954	0.872
0.8	0.25	40	CLT	0.15	0.112	0.153	0.153	0.934	0.815
0.8	0.30	38	CLT	0.15	0.103	0.109	0.109	0.931	0.783
0.9	0.05	50	CLT	0.11	0.137	0.170	0.301	0.968	0.915
0.9	0.10	43	CLT	0.12	0.134	0.183	0.382	0.958	0.883
0.9	0.25	32	CLT	0.15	0.110	0.161	0.161	0.936	0.818
0.9	0.30	31	CLT	0.15	0.105	0.127	0.127	0.940	0.835
1.0	0.05	41	CLT	0.11	0.136	0.163	0.294	0.967	0.910
1.0	0.10	35	CLT	0.12	0.135	0.201	0.365	0.963	0.906
1.0	0.25	26	CLT	0.15	0.114	0.168	0.168	0.938	0.837
1.0	0.30	25	CLT	0.15	0.105	0.102	0.102	0.937	0.835
1.1	0.05	34	CLT	0.11	0.139	0.171	0.311	0.970	0.923
1.1	0.10	29	CLT	0.12	0.134	0.185	0.378	0.964	0.908
1.1	0.25	22	CLT	0.15	0.112	0.166	0.166	0.943	0.846
1.1	0.30	21	CLT	0.15	0.104	0.101	0.101	0.941	0.849

Table 43: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=500, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.11	0.141	0.182	0.333	0.869	0.537
0.7	0.10	51	CLT	0.12	0.134	0.183	0.382	0.856	0.477
0.7	0.25	42	CLT	0.15	0.111	0.165	0.165	0.867	0.398
0.7	0.30	40	CLT	0.15	0.106	0.109	0.109	0.863	0.357
0.8	0.05	45	CLT	0.11	0.135	0.177	0.313	0.855	0.502
0.8	0.10	39	CLT	0.12	0.130	0.176	0.338	0.850	0.485
0.8	0.25	33	CLT	0.15	0.112	0.172	0.172	0.876	0.439
0.8	0.30	31	CLT	0.15	0.103	0.104	0.104	0.869	0.363
0.9	0.05	36	CLT	0.11	0.143	0.195	0.355	0.869	0.540
0.9	0.10	32	CLT	0.12	0.138	0.206	0.401	0.867	0.521
0.9	0.25	26	CLT	0.15	0.112	0.171	0.171	0.876	0.437
0.9	0.30	25	CLT	0.15	0.104	0.101	0.101	0.883	0.488
1.0	0.05	30	CLT	0.11	0.141	0.178	0.328	0.879	0.571
1.0	0.10	26	CLT	0.12	0.131	0.180	0.354	0.875	0.541
1.0	0.25	21	CLT	0.15	0.116	0.197	0.197	0.871	0.402
1.0	0.30	20	CLT	0.15	0.104	0.113	0.113	0.870	0.380
1.1	0.05	25	CLT	0.11	0.143	0.164	0.331	0.879	0.557
1.1	0.10	22	CLT	0.12	0.133	0.196	0.357	0.888	0.598
1.1	0.25	18	CLT	0.15	0.112	0.157	0.157	0.883	0.504
1.1	0.30	17	CLT	0.15	0.107	0.123	0.123	0.881	0.462

Table 44: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=500, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	100	CLT	0.06	0.134	0.144	0.335	0.982	0.944
0.7	0.10	84	CLT	0.07	0.131	0.154	0.379	0.983	0.936
0.7	0.25	60	CLT	0.12	0.114	0.174	0.266	0.958	0.861
0.7	0.30	56	CLT	0.13	0.100	0.155	0.208	0.958	0.869
0.8	0.05	77	CLT	0.06	0.145	0.162	0.354	0.979	0.945
0.8	0.10	65	CLT	0.07	0.131	0.165	0.381	0.986	0.944
0.8	0.25	46	CLT	0.12	0.107	0.155	0.245	0.964	0.886
0.8	0.30	43	CLT	0.13	0.109	0.195	0.245	0.959	0.871
0.9	0.05	62	CLT	0.06	0.143	0.150	0.337	0.982	0.947
0.9	0.10	51	CLT	0.07	0.132	0.157	0.391	0.986	0.937
0.9	0.25	37	CLT	0.12	0.112	0.196	0.268	0.961	0.872
0.9	0.30	34	CLT	0.13	0.107	0.194	0.244	0.956	0.859
1.0	0.05	51	CLT	0.06	0.144	0.156	0.355	0.989	0.967
1.0	0.10	42	CLT	0.07	0.127	0.158	0.369	0.983	0.932
1.0	0.25	30	CLT	0.12	0.113	0.189	0.277	0.964	0.901
1.0	0.30	28	CLT	0.13	0.103	0.174	0.214	0.959	0.878
1.1	0.05	42	CLT	0.06	0.141	0.146	0.357	0.989	0.973
1.1	0.10	35	CLT	0.07	0.138	0.155	0.401	0.985	0.942
1.1	0.25	25	CLT	0.12	0.110	0.177	0.264	0.967	0.907
1.1	0.30	23	CLT	0.13	0.103	0.179	0.220	0.957	0.862

Table 45: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=100, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	CLT	0.06	0.133	0.000	0.326	0.857	0.591
0.7	0.10	51	CLT	0.07	0.131	0.136	0.386	0.865	0.543
0.7	0.25	42	CLT	0.12	0.109	0.165	0.268	0.864	0.520
0.7	0.30	40	CLT	0.13	0.103	0.178	0.234	0.856	0.491
0.8	0.05	45	CLT	0.06	0.134	0.000	0.336	0.851	0.581
0.8	0.10	39	CLT	0.07	0.139	0.158	0.389	0.855	0.529
0.8	0.25	33	CLT	0.12	0.114	0.187	0.287	0.877	0.566
0.8	0.30	31	CLT	0.13	0.104	0.179	0.233	0.860	0.514
0.9	0.05	36	CLT	0.06	0.133	0.000	0.321	0.879	0.645
0.9	0.10	32	CLT	0.07	0.129	0.142	0.373	0.877	0.579
0.9	0.25	26	CLT	0.12	0.110	0.175	0.277	0.871	0.555
0.9	0.30	25	CLT	0.13	0.106	0.168	0.229	0.872	0.560
1.0	0.05	30	CLT	0.06	0.139	0.000	0.358	0.872	0.622
1.0	0.10	26	CLT	0.07	0.137	0.164	0.391	0.868	0.563
1.0	0.25	21	CLT	0.12	0.116	0.208	0.295	0.864	0.527
1.0	0.30	20	CLT	0.13	0.103	0.167	0.216	0.870	0.541
1.1	0.05	25	CLT	0.06	0.139	0.000	0.337	0.873	0.628
1.1	0.10	22	CLT	0.07	0.130	0.159	0.359	0.882	0.605
1.1	0.25	18	CLT	0.12	0.111	0.186	0.278	0.881	0.582
1.1	0.30	17	CLT	0.13	0.099	0.151	0.203	0.871	0.532

Table 46: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=100, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	120	Rom		0.143	0.120	0.332	0.916	0.912
0.7	0.10	104	Rom		0.133	0.098	0.373	0.992	0.983
0.7	0.25	67	CLT	0.10	0.106	0.161	0.277	0.977	0.928
0.7	0.30	62	CLT	0.11	0.107	0.172	0.260	0.970	0.905
0.8	0.05	92	Rom		0.133	0.122	0.309	0.922	0.919
0.8	0.10	81	Rom		0.132	0.095	0.365	0.995	0.988
0.8	0.25	52	CLT	0.10	0.116	0.177	0.326	0.982	0.947
0.8	0.30	48	CLT	0.11	0.106	0.184	0.271	0.974	0.915
0.9	0.05	74	Rom		0.141	0.125	0.313	0.912	0.911
0.9	0.10	64	Rom		0.131	0.095	0.369	0.989	0.969
0.9	0.25	41	CLT	0.10	0.113	0.175	0.307	0.973	0.917
0.9	0.30	38	CLT	0.11	0.102	0.161	0.262	0.974	0.916
1.0	0.05	60	Rom		0.143	0.118	0.324	0.908	0.907
1.0	0.10	53	Rom		0.128	0.092	0.365	0.995	0.987
1.0	0.25	34	CLT	0.10	0.113	0.168	0.303	0.981	0.933
1.0	0.30	31	CLT	0.11	0.104	0.181	0.270	0.974	0.921
1.1	0.05	50	Rom		0.131	0.106	0.303	0.929	0.926
1.1	0.10	44	Rom		0.132	0.084	0.363	0.994	0.981
1.1	0.25	28	CLT	0.10	0.112	0.179	0.314	0.980	0.943
1.1	0.30	26	CLT	0.11	0.099	0.166	0.261	0.977	0.932

Table 47: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=50, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	Rom		0.124	0.113	0.278	0.800	0.707
0.7	0.10	51	Rom		0.131	0.123	0.366	0.860	0.582
0.7	0.25	42	CLT	0.10	0.111	0.171	0.295	0.850	0.516
0.7	0.30	40	CLT	0.11	0.108	0.160	0.279	0.853	0.510
0.8	0.05	45	Rom		0.142	0.112	0.311	0.797	0.700
0.8	0.10	39	Rom		0.126	0.096	0.350	0.855	0.584
0.8	0.25	33	CLT	0.10	0.107	0.151	0.290	0.871	0.571
0.8	0.30	31	CLT	0.11	0.111	0.185	0.298	0.864	0.568
0.9	0.05	36	Rom		0.145	0.139	0.296	0.792	0.695
0.9	0.10	32	Rom		0.126	0.105	0.365	0.884	0.658
0.9	0.25	26	CLT	0.10	0.116	0.163	0.311	0.869	0.568
0.9	0.30	25	CLT	0.11	0.100	0.153	0.243	0.872	0.556
1.0	0.05	30	Rom		0.142	0.135	0.306	0.831	0.728
1.0	0.10	26	Rom		0.125	0.094	0.348	0.877	0.625
1.0	0.25	21	CLT	0.10	0.106	0.149	0.285	0.873	0.585
1.0	0.30	20	CLT	0.11	0.104	0.168	0.260	0.858	0.514
1.1	0.05	25	Rom		0.135	0.120	0.313	0.833	0.742
1.1	0.10	22	Rom		0.144	0.106	0.390	0.870	0.661
1.1	0.25	18	CLT	0.10	0.107	0.151	0.267	0.879	0.583
1.1	0.30	17	CLT	0.11	0.103	0.168	0.279	0.873	0.574

Table 48: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=50, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	132	Rom		0.126	0.109	0.215	0.649	0.649
0.7	0.10	115	Rom		0.135	0.139	0.322	0.891	0.891
0.7	0.25	91	Rom		0.107	0.085	0.331	0.993	0.981
0.7	0.30	86	Rom		0.108	0.078	0.320	0.996	0.980
0.8	0.05	102	Rom		0.142	0.136	0.237	0.644	0.644
0.8	0.10	88	Rom		0.122	0.119	0.278	0.873	0.872
0.8	0.25	70	Rom		0.113	0.090	0.352	0.994	0.983
0.8	0.30	66	Rom		0.099	0.065	0.293	0.994	0.975
0.9	0.05	81	Rom		0.127	0.112	0.215	0.621	0.621
0.9	0.10	70	Rom		0.130	0.125	0.288	0.880	0.880
0.9	0.25	56	Rom		0.109	0.096	0.340	0.994	0.983
0.9	0.30	53	Rom		0.107	0.082	0.319	0.997	0.988
1.0	0.05	66	Rom		0.121	0.104	0.207	0.621	0.621
1.0	0.10	58	Rom		0.123	0.108	0.287	0.850	0.849
1.0	0.25	46	Rom		0.105	0.094	0.313	0.997	0.992
1.0	0.30	43	Rom		0.105	0.092	0.313	0.996	0.982
1.1	0.05	55	Rom		0.133	0.118	0.221	0.648	0.648
1.1	0.10	48	Rom		0.127	0.112	0.289	0.886	0.886
1.1	0.25	38	Rom		0.110	0.110	0.330	0.994	0.980
1.1	0.30	36	Rom		0.112	0.082	0.321	0.995	0.979

Table 49: Sample size determined via tp-TPP power, under AutFDP control, $\alpha = 0.15$, m=20, $\rho = 0.2$

es	p_1	n ac	Aut	a st	FDR	t FDP a	t FDP	AvgP	t TPP
0.7	0.05	59	Rom		0.131	0.114	0.228	0.603	0.582
0.7	0.10	51	Rom		0.145	0.149	0.291	0.746	0.677
0.7	0.25	42	Rom		0.115	0.109	0.336	0.852	0.602
0.7	0.30	40	Rom		0.105	0.088	0.319	0.855	0.536
0.8	0.05	45	Rom		0.130	0.125	0.210	0.577	0.563
0.8	0.10	39	Rom		0.128	0.120	0.278	0.745	0.664
0.8	0.25	33	Rom		0.111	0.101	0.322	0.871	0.615
0.8	0.30	31	Rom		0.104	0.091	0.300	0.869	0.583
0.9	0.05	36	Rom		0.127	0.113	0.211	0.601	0.584
0.9	0.10	32	Rom		0.124	0.110	0.269	0.767	0.700
0.9	0.25	26	Rom		0.104	0.091	0.306	0.865	0.597
0.9	0.30	25	Rom		0.108	0.096	0.308	0.866	0.564
1.0	0.05	30	Rom		0.136	0.127	0.222	0.593	0.577
1.0	0.10	26	Rom		0.138	0.141	0.302	0.758	0.699
1.0	0.25	21	Rom		0.114	0.105	0.332	0.846	0.602
1.0	0.30	20	Rom		0.106	0.090	0.297	0.857	0.567
1.1	0.05	25	Rom		0.138	0.131	0.228	0.566	0.550
1.1	0.10	22	Rom		0.126	0.123	0.282	0.804	0.746
1.1	0.25	18	Rom		0.113	0.112	0.338	0.875	0.630
1.1	0.30	17	Rom		0.105	0.086	0.296	0.871	0.589

Table 50: Sample size determined via average power, under BHFDR control, $\alpha = 0.15$, m=20, $\rho = 0.2$

Overview

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- Even for adequately powered studies, the dispersion of the *FDP* warrants control of its right tail probability rather than its mean for situations of *fewer* than 2000 simultaneous tests
- Sample sizes should be derived using the tail probability based tp-TPP power rather than the average power.

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- Simulation suggests that
    - all of the FDP control methods control the promised characteristic at the nominal level
    - Both the population mean based average power and the CLT based tp-TPP power perform well under all of the FDP control methods
    - When $mp_1 < 5$ the performance deteriorates greatly, but this is a non-sensical design.

What about Bonferroni

- If desired expected number of true positives, $m\pi_1$, is 3 or less then use Bonferroni

Thanks

- Thank you for your time and attention

Thanks

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- Questions/Comments?