

Package ‘Langevin’

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Type Package

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Langevin-package *An R package for stochastic data analysis*

Description

The **Langevin** package provides functions to estimate drift and diffusion functions from data sets.

Details

This package was developed by the research group *Turbulence, Wind energy and Stochastics* (TWiSt) at the Carl von Ossietzky University of Oldenburg (Germany).

Mathematical Background

A wide range of dynamic systems can be described by a stochastic differential equation, the Langevin equation. The time derivative of the system trajectory $\dot{X}(t)$ can be expressed as a sum of a deterministic part $D^{(1)}$ and the product of a stochastic force $\Gamma(t)$ and a weight coefficient $D^{(2)}$. The stochastic force $\Gamma(t)$ is δ -correlated Gaussian white noise.

For stationary continuous Markov processes Siegert et al. and Friedrich et al. developed a method to reconstruct drift $D^{(1)}$ and diffusion $D^{(2)}$ directly from measured data.

$$\dot{X}(t) = D^{(1)}(X(t), t) + \sqrt{D^{(2)}(X(t), t)} \Gamma(t) \quad \text{with}$$

$$D^{(n)}(x, t) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} M^{(n)}(x, t, \tau) \quad \text{and}$$

$$M^{(n)}(x, t, \tau) = \frac{1}{n!} \langle (X(t + \tau) - x)^n \rangle_{|X(t)=x}$$

The Langevin equation should be interpreted in the way that for every time t_i where the system meets an arbitrary but fixed point x in phase space, $X(t_i + \tau)$ is defined by the deterministic function $D^{(1)}(x)$ and the stochastic function $\sqrt{D^{(2)}(x)}\Gamma(t_i)$. Both, $D^{(1)}(x)$ and $D^{(2)}(x)$ are constant for fixed x .

One can integrate drift and diffusion numerically over small intervals. If the system is at time t in the state $x = X(t)$ the drift can be calculated for small τ by averaging over the difference of the system state at $t + \tau$ and the state at t . The average has to be taken over the whole ensemble or in the stationary case over all $t = t_i$ with $X(t_i) = x$. Diffusion can be calculated analogously.

Author(s)

Philip Rinn

References

A review of the Langevin method can be found at:

Friedrich, R.; et al. (2011) *Approaching Complexity by Stochastic Methods: From Biological Systems to Turbulence*. Physics Reports, 506(5), 87–162.

For further reading:

Risken, H. (1996) *The Fokker-Planck equation*. Springer.

Siebert, S.; et al. (1998) *Analysis of data sets of stochastic systems*. Phys. Lett. A.

Friedrich, R.; et al. (2000) *Extracting model equations from experimental data*. Phys. Lett. A.

Honisch, C.; Friedrich, R. (2011). *Estimation of Kramers-Moyal coefficients at low sampling rates..* Physical Review E, 83(6), 066701.

Langevin1D	<i>Calculate the Drift and Diffusion of one-dimensional stochastic processes</i>
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Description

Langevin1D calculates the Drift and Diffusion vectors (with errors) for a given time series.

Usage

```
Langevin1D(
    data,
    bins,
    steps,
    sf = ifelse(is.ts(data), frequency(data), 1),
    bin_min = 100,
    reqThreads = -1,
    kernel = FALSE,
    h
)
```

Arguments

data	a vector containing the time series or a time-series object.
bins	a scalar denoting the number of bins to calculate the conditional moments on.
steps	a vector giving the τ steps to calculate the conditional moments (in samples ($=\tau * sf$)). Only used if kernel is FALSE.
sf	a scalar denoting the sampling frequency (optional if data is a time-series object).
bin_min	a scalar denoting the minimal number of events per bin. Defaults to 100.
reqThreads	a scalar denoting how many threads to use. Defaults to -1 which means all available cores. Only used if kernel is FALSE.

kernel	a logical denoting if the kernel based Nadaraya-Watson estimator should be used to calculate drift and diffusion vectors.
h	a scalar denoting the bandwidth of the data. Defaults to Scott's variation of Silverman's rule of thumb. Only used if kernel is TRUE.

Value

Langevin1D returns a list with thirteen (six if kernel is TRUE) components:

D1	a vector of the Drift coefficient for each bin.
eD1	a vector of the error of the Drift coefficient for each bin.
D2	a vector of the Diffusion coefficient for each bin.
eD2	a vector of the error of the Diffusion coefficient for each bin.
D4	a vector of the fourth Kramers-Moyal coefficient for each bin.
mean_bin	a vector of the mean value per bin.
density	a vector of the number of events per bin. If kernel is FALSE.
M1	a matrix of the first conditional moment for each τ . Rows correspond to bin, columns to τ . If kernel is FALSE.
eM1	a matrix of the error of the first conditional moment for each τ . Rows correspond to bin, columns to τ . If kernel is FALSE.
M2	a matrix of the second conditional moment for each τ . Rows correspond to bin, columns to τ . If kernel is FALSE.
eM2	a matrix of the error of the second conditional moment for each τ . Rows correspond to bin, columns to τ . If kernel is FALSE.
M4	a matrix of the fourth conditional moment for each τ . Rows correspond to bin, columns to τ . If kernel is FALSE.
U	a vector of the bin borders. If kernel is FALSE.

Author(s)

Philip Rinn

See Also

[Langevin2D](#)

Examples

```
# Set number of bins, steps and the sampling frequency
bins <- 20
steps <- c(1:5)
sf <- 1000

#### Linear drift, constant diffusion
```

```

# Generate a time series with linear D^1 = -x and constant D^2 = 1
x <- timeseries1D(N = 1e6, d11 = -1, d20 = 1, sf = sf)
# Do the analysis
est <- Langevin1D(data = x, bins = bins, steps = steps, sf = sf)
# Plot the result and add the theoretical expectation as red line
plot(est$mean_bin, est$D1)
lines(est$mean_bin, -est$mean_bin, col = "red")
plot(est$mean_bin, est$D2)
abline(h = 1, col = "red")

#### Cubic drift, constant diffusion

# Generate a time series with cubic D^1 = x - x^3 and constant D^2 = 1
x <- timeseries1D(N = 1e6, d13 = -1, d11 = 1, d20 = 1, sf = sf)
# Do the analysis
est <- Langevin1D(data = x, bins = bins, steps = steps, sf = sf)
# Plot the result and add the theoretical expectation as red line
plot(est$mean_bin, est$D1)
lines(est$mean_bin, est$mean_bin - est$mean_bin^3, col = "red")
plot(est$mean_bin, est$D2)
abline(h = 1, col = "red")

```

Langevin2D

Calculate the Drift and Diffusion of two-dimensional stochastic processes

Description

Langevin2D calculates the Drift (with error) and Diffusion matrices for given time series.

Usage

```

Langevin2D(
  data,
  bins,
  steps,
  sf = ifelse(is.mts(data), frequency(data), 1),
  bin_min = 100,
  reqThreads = -1
)

```

Arguments

data	a matrix containing the time series as columns or a time-series object.
bins	a scalar denoting the number of bins to calculate Drift and Diffusion on.
steps	a vector giving the τ steps to calculate the moments (in samples).
sf	a scalar denoting the sampling frequency (optional if data is a time-series object).

bin_min	a scalar denoting the minimal number of events per bin. Defaults to 100.
reqThreads	a scalar denoting how many threads to use. Defaults to -1 which means all available cores.

Value

Langevin2D returns a list with nine components:

D1	a tensor with all values of the drift coefficient. Dimension is bins x bins x 2. The first bins x bins elements define the drift $D_1^{(1)}$ for the first variable and the rest define the drift $D_2^{(1)}$ for the second variable.
eD1	a tensor with all estimated errors of the drift coefficient. Dimension is bins x bins x 2. Same expression as above.
D2	a tensor with all values of the diffusion coefficient. Dimension is bins x bins x 3. The first bins x bins elements define the diffusion $D_{11}^{(2)}$, the second bins x bins elements define the diffusion $D_{22}^{(2)}$ and the rest define the diffusion $D_{12}^{(2)} = D_{21}^{(2)}$.
mean_bin	a matrix of the mean value per bin. Dimension is bins x bins x 2. The first bins x bins elements define the mean for the first variable and the rest for the second variable.
density	a matrix of the number of events per bin. Rows label the bin of the first variable and columns the second variable.
M1	a tensor of the first moment for each bin (line label) and each τ step (column label). Dimension is bins x bins x 2length(steps).
eM1	a tensor of the standard deviation of the first moment for each bin (line label) and each τ step (column label). Dimension is bins x bins x 2length(steps).
M2	a tensor of the second moment for each bin (line label) and each τ step (column label). Dimension is bins x bins x 3length(steps).
U	a matrix of the bin borders

Author(s)

Philip Rinn

See Also

[Langevin1D](#)

plot.Langevin	<i>Plot estimated drift and diffusion coefficients</i>
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Description

plot method for class "Langevin". This method is only implemented for one-dimensional analysis for now.

Usage

```
## S3 method for class 'Langevin'  
plot(x, pch = 20, ...)
```

Arguments

x	an object of class "Langevin".
pch	Either an integer specifying a symbol or a single character to be used as the default in plotting points. See points for possible values and their interpretation. Default is pch = 20.
...	Arguments to be passed to methods, such as par .

Author(s)

Philip Rinn

print.Langevin	<i>Print estimated drift and diffusion coefficients</i>
----------------	---

Description

print method for class "Langevin".

Usage

```
## S3 method for class 'Langevin'  
print(x, digits = max(3, getOption("digits") - 3), ...)
```

Arguments

x	an object of class "Langevin".
digits	integer, used for number formatting with signif() .
...	further arguments to be passed to or from other methods. They are ignored in this function.

Value

The function `print.Langevin()` returns an overview of the estimated drift and diffusion coefficients.

Author(s)

Philip Rinn

summary.Langevin	<i>Summarize estimated drift and diffusion coefficients</i>
------------------	---

Description

summary method for class "Langevin".

Usage

```
## S3 method for class 'Langevin'
summary(object, ..., digits = max(3, getOption("digits") - 3))
```

Arguments

object	an object of class "Langevin".
...	arguments to be passed to or from other methods. They are ignored in this function.
digits	integer, used for number formatting with <code>signif()</code> .

Value

The function `summary.Langevin()` returns a summary of the estimated drift and diffusion coefficients

Author(s)

Philip Rinn

timeseries1D *Generate a 1D Langevin process*

Description

timeseries1D generates a one-dimensional Langevin process using a simple Euler integration. The drift function is a cubic polynomial, the diffusion function a quadratic.

Usage

```
timeseries1D(  
  N,  
  startpoint = 0,  
  d13 = 0,  
  d12 = 0,  
  d11 = -1,  
  d10 = 0,  
  d22 = 0,  
  d21 = 0,  
  d20 = 1,  
  sf = 1000,  
  dt = 0  
)
```

Arguments

N	a scalar denoting the length of the time-series to generate.
startpoint	a scalar denoting the starting point of the time series.
d13, d12, d11, d10	scalars denoting the coefficients for the drift polynomial.
d22, d21, d20	scalars denoting the coefficients for the diffusion polynomial.
sf	a scalar denoting the sampling frequency.
dt	a scalar denoting the maximal time step of integration. Default dt=0 yields dt=1/sf.

Value

timeseries1D returns a time-series object of length N with the generated time-series.

Author(s)

Philip Rinn

See Also

[timeseries2D](#)

Examples

```
# Generate standardized Ornstein-Uhlenbeck-Process (d11=-1, d20=1)
# with integration time step 0.01 and sampling frequency 1
s <- timeseries1D(N=1e4, sf=1, dt=0.01);
t <- 1:1e4;
plot(t, s, t="1", main=paste("mean:", mean(s), " var:", var(s)));
```

timeseries2D

*Generate a 2D Langevin process***Description**

timeseries2D generates a two-dimensional Langevin process using a simple Euler integration. The drift function is a cubic polynomial, the diffusion function a quadratic.

Usage

```
timeseries2D(
  N,
  startpointx = 0,
  startpointy = 0,
  D1_1 = matrix(c(0, -1, rep(0, 14)), nrow = 4),
  D1_2 = matrix(c(0, 0, 0, 0, -1, rep(0, 11)), nrow = 4),
  g_11 = matrix(c(1, 0, 0, 0, 0, 0, 0, 0, 0), nrow = 3),
  g_12 = matrix(c(0, 0, 0, 0, 0, 0, 0, 0, 0), nrow = 3),
  g_21 = matrix(c(0, 0, 0, 0, 0, 0, 0, 0, 0), nrow = 3),
  g_22 = matrix(c(1, 0, 0, 0, 0, 0, 0, 0, 0), nrow = 3),
  sf = 1000,
  dt = 0
)
```

Arguments

N	a scalar denoting the length of the time-series to generate.
startpointx	a scalar denoting the starting point of the time series x.
startpointy	a scalar denoting the starting point of the time series y.
D1_1	a 4x4 matrix denoting the coefficients of D1 for x.
D1_2	a 4x4 matrix denoting the coefficients of D1 for y.
g_11	a 3x3 matrix denoting the coefficients of g11 for x.
g_12	a 3x3 matrix denoting the coefficients of g12 for x.
g_21	a 3x3 matrix denoting the coefficients of g21 for y.
g_22	a 3x3 matrix denoting the coefficients of g22 for y.
sf	a scalar denoting the sampling frequency.
dt	a scalar denoting the maximal time step of integration. Default dt=0 yields dt=1/sf.

Details

The elements a_{ij} of the matrices are defined by the corresponding equations for the drift and diffusion terms:

$$D_{1,2}^1 = \sum_{i,j=1}^4 a_{ij} x_1^{(i-1)} x_2^{(j-1)}$$

with $a_{ij} = 0$ for $i + j > 5$.

$$g_{11,12,21,22} = \sum_{i,j=1}^3 a_{ij} x_1^{(i-1)} x_2^{(j-1)}$$

with $a_{ij} = 0$ for $i + j > 4$

Value

`timeseries2D` returns a time-series object with the generated time-series as columns.

Author(s)

Philip Rinn

See Also

[timeseries1D](#)

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